Economics, Department of

Economics Theses

University of Puget Sound *Year* 2010

Optimal Portfolio Selection of Credit Unions and the Probability of Failure

J. Zack Keim
University of Puget Sound

Optimal Portfolio Selection of Credit Unions and the Probability of Failure

J. Zack Keim
Fall 2010

Senior thesis submitted in partial fulfillment
of the requirements for a
Bachelor of Science degree in Economics
at the University of Puget Sound

Optimal Portfolio Selection of Credit Unions and the Probability of Failure

Introduction

Credit Unions constitute a significant portion of the financial intermediary sector. As such, credit unions have considerable influence on the economy as a whole. Given this influence, credit union failures pose a serious risk to the well being of the financial sector as well as the real economy. Therefore, knowing the conditions that increase the likelihood of credit union failure is essential. Although there is a significant amount of literature devoted to analyzing the probability of for-profit financial intermediary failure, both empirically and theoretically, there has yet to be any exploration into a theory of credit union failure. Moreover, the existing theoretical framework describing credit union behavior fails to take into account the stochastic nature of the environment in which credit unions operate. The purpose of this thesis is twofold: first to present an adequate theory that can be used to explain the credit union decision making process in an uncertain operating environment and second to use that framework to establish what individual credit union characteristics will increase the likelihood of failure.

Because for-profit enterprises with market power attempt to maximize profits at the expense of consumers, markets characterized by such conditions often do not achieve the socially optimal outcome. Furthermore, there exists asymmetric information in lending markets, in that lenders do not know borrowers' true willingness to repay. Thus, borrowers can often take advantage of lenders by defaulting on loans. The effect this has on the lending market is that often lenders will restrict credit to only individuals who can show an almost certain willingness and ability to repay. This contract failure results in a non-welfare maximizing outcome, as individuals capable of not defaulting are often barred from accessing credit. As argued in Hansmann (1987), non-profit institutions may be best at alleviating these market failures.

Credit unions have arisen as a way to overcome these market failures. Credit unions are cooperative institutions in which members are the owners and users of the credit union's services. By linking the owners and users of services, credit unions are able to remove the asymmetries in lending, since the benefactor of default is now the bearer of its cost. The alleviation of this contract failure allows credit unions to lend at lower rates, since there is a lower likelihood of default, and thus, less needed compensation for risk. This also allows credit unions to lend to borrowers who may not have access to credit from traditional for-profit institutions. In addition, because credit unions are bound by the non-distribution constraint, they cannot retain any profits, which means, all excess revenue is distributed to depositors at the credit union or back to borrowers via interest rate refunds. This serves to further lower the rates charged on loans, and increases the rates paid on deposits. Furthermore, as pointed out in Hansmann (1987), by not having a profit motive, credit unions, and really all non-profit firms for that matter, do not seek to "monopolistically exploit (their) patrons", or in other words, charge a price above the social optimum. This also contributes significantly to lower loan rates and higher deposit rates.

The distinct behavior of credit unions results in credit unions acting differently than would be expected when presented with various environmental changes than for-profit financial intermediaries. For instance, the distinct characteristics of credit unions, like the common bond requirement, cause credit unions to have unique structural features. These distinct structural characteristics and environmental factors have a direct effect on credit union behavior. In some situations, these idiosyncratic characteristics and environmental factors can cause credit unions to make decisions that will increase the probability of them failing. The purpose of this thesis is to present a new model for credit union decision making under uncertainty and to determine how changes in costs, systematic risk, and degree of competition effect the optimal portfolio allocation of credit unions, and how that may increase or decrease the likelihood of failure. The presentation will be as follows: i) a review of the literature, ii) derivation of the efficiency frontier for credit unions, iii) discussion of the utility function for credit unions and the derivation of the unique portfolio iv) effects of various characteristics on the probability of credit union failure, and v) conclusions.

Review of Literature

Perhaps the most significant work analyzing credit union behavior is that of Smith, Cargill, and Meyer (1981), Smith (1984), and Smith (1988). Since Smith (1988), theories on credit union decision making have laid dormant. Only Smith (1988) has performed an analysis on the likelihood of credit union failure. None of the above literature has analyzed credit union decision making from the perspective of modern portfolio theory. However, in contrast to the minimal amount of research pertaining to credit union decision making, the literature on for-profit financial intermediary behavior is rather rich. In regards to these institutions, it has been the norm to employ modern portfolio theory as the analytical mechanism to describe behavior, as has been done in Blair and Heggestad (1978), Hart and Jaffee (1974), Klein (1971), Koehn and Santomero (1980) and Sealey Jr. (1980).

Smith, Cargill, and Meyer (1981) and Smith (1984) were among the first to describe credit unions not as profit or revenue maximizing institutions, but as benefit maximizers. The model laid out in Smith, Cargill, and Meyer (1981) describes credit union behavior in the riskless setting. Credit unions are viewed as institutions that seek to maximize net benefits derived from the use of its lending and depositing services. As is much the tradition in the literature on credit unions (see Smith (1984) and Smith (1988)), benefits are described as the difference between the market rate and that of the credit unions. This model incorporates the inherent saver versus borrower conflict by weighting the respective contributions of loans and savings to benefits. Smith (1984) extends this model to an inter-temporal setting, by redefining benefits as the present value of all future benefits.

However, the riskless setting assumed in Smith, Cargill, and Meyer (1981) and Smith (1984) omits the characteristic uncertainty present in the operating environment of credit unions. To capture this environmental characteristic, Smith (1988) models the objective function by extending Smith (1984) to include an environment in which the returns on investments and loans are unknown at the beginning of the period, and then known ex post. To model this randomness, Smith subjects the reserve ratio to a chance constraint in which a minimum reserve requirement must be meet at the end of each period. To meet this reserve requirement, which may be negative as a result of an increase in the number of loan defaults, repayment delinquencies, or negative return on capital market investments a credit union will have to increase the amount it borrows in financial markets. Although this stylized approach to modeling decisions under uncertainty presents detailed descriptions of credit union behavior, and presents intuitively correct comparative statics; its ability to describe the portfolio of a credit union and how other factors can increase the likelihood that the reserve requirement is not met is limited.

Extending the model presented in Smith, Cargill, and Meyer (1981), Rahman and McNeil (1999) model the behavior of large U.S. credit unions under decisions of uncertainty. The stochastic nature of the operating environment for credit unions is significantly different than that of Smith (1988). In Rahman and McNeil, the levels of deposits are viewed as random, and are incorporated via the utility function of the credit union. The goal of the credit union in this model is to maximize utility, where utility is a function of income and deposits, subject to a balance sheet constraint. The return on loans, securities, the savings rate, reserve ratio, and capital reserves are all viewed as exogenous and non stochastic. For one, this model fails to describe the uncertainty of returns on loans and securities, which is a fundamentally inherent aspect of these variables. Secondly, the utility function gives no consideration to the benefits that could be had from lending. Together, these two flaws in the model greatly limit its explanatory power.

Although the literature pertaining to credit union decision making under uncertainty is minimal, the amount of literature regarding for-profit financial intermediaries is quit rich. As with the models of credit union behavior there are two ways to describe for-profit financial intermediary behavior—portfolio theory and constrained optimization in the riskless environment. Given the inherent randomness associated with financial markets, it appears more appropriate to use models of behavior that incorporate this aspect.

Hart and Jaffee use portfolio theory to describe the behavior of financial intermediaries by modifying the existing framework of portfolio theory laid out by Markowitz (1952) and Merton (1972). Hart and Jaffee’s model alters and applies several assumptions that allow for the derivation of the efficiency frontier for financial intermediaries and for comparative static analysis of its properties. This model is efficient in providing a minimal framework in which further theory can be built upon. Sealey adds further to this model by altering the assumptions underlying the model. Sealey assumes that deposit rates are determined by financial institutions, and that the supply of deposits are stochastically determined period to period.

In terms of theory that examines the probability of failure of financial intermediaries, portfolio theory appears to be the dominant analytical device as has been the case in Blair and Heggestad (1978) and Koehn and Santomero (1980). The benefit of portfolio theory in examining the probability of failure is that it is easier to decipher what conditions would lead a financial intermediary to engage in more risk taking, which would increase the likelihood of failure. For example, Blair and Heggestad's exploration into the effects of regulation and the probability of retail bank failure shows that increased risk taking leads to a higher probability of failure. The frame work used by Blair and Heggestad was developed by Roy (1952) and Pyle and Turnovsky (1970). Koehn and Santomero proceeded Blair and Heggestad in a similar analysis. Both Koehn and Santomero and Blair and Heggestad showed that the portfolio theory when applied to financial intermediaries could be altered and modified in order to theoretically analyze the effects of regulatory changes in the environment in which these institutions operated, and from that the probability of failure could be attained.

Even though many of theories on the probability of bank failure and decisions under uncertainty are robust, their use is inapplicable in an analysis of the probability of credit union failure. The reason underlying this is the differences in the objectives of for-profit financial institutions and that of credit unions.

The Model

Given the inherent stochastic nature of loan repayment and deposit with-drawl, and the decision faced by credit unions of whom to lend to, it seems fitting to apply modern portfolio theory to describe credit union behavior. However, credit unions do not behave as for-profit depositories do, and, as a result of this, the existing framework of modern portfolio theory must be altered in order for it to be applicable to credit unions. Perhaps the most important aspect that must be included in the model is the benefits accrued by members, since the objective of a credit union is to maximize these benefits. Another idiosyncratic characteristic that applies solely to credit unions, and has not been discussed in the literature on portfolio theory, is the non-distribution constraint. Including this constraint requires limiting the amount of income that credit unions will earn from a portfolio to zero. Finally, credit unions, unlike their for-profit depository counter parts, are limited to whom they can do business with by a common bond requirement. Thus, credit unions will not have access to the wide array potential borrowers that for-profit depositories do. In applying portfolio theory to describe credit union behavior, it is clear that these aspects must be considered in full.

The Efficiency Frontier for Credit Unions

To begin the discussion of the efficiency frontier, it is first necessary to define the portfolio of a credit union. The portfolio of a credit union in its simplest form consists solely of loans and deposits of the credit union. Because of the common bond requirement, the quantity of different loans, or individuals in which the credit union can lend, is limited. Likewise, the quantity of individuals who can deposit at savings at a credit union is limited. To describe who can and cannot do business with a credit union, I define the membership base as all individuals that satisfy the common bond requirement. In other words, if n denotes the number of individuals who are in the membership base of a credit union, then the credit union will have n distinct individuals in which it can lend too. And by that same argument, n individuals in which it can take deposits from. Given the n individuals in the membership base, the percentage of the total loans of the credit union lent to the i^{th} individual can be described by x_{Li}. Loans, in this model, are viewed as positive assets, i.e. $x_{Li} \geq 0$ for all $i = 1, 2, \dots, n$. For all n members, the quantity of loans and deposits held by all members can be summarized by the $1 \times n$ component vector $\boldsymbol{x}$.

Since all the components of $\boldsymbol{x}$ are percentages of total loans, the sum of all the components in $\boldsymbol{x}$ must equal 1. That is,

$$1 = \boldsymbol{x}^t \mathbf{1}.$$

In order to make portfolio theory applicable in describing credit union behavior the quantity of benefits generated from a set of assets and liabilities must be incorporated. There are a variety of ways to view and quantify the benefits produced by credit unions. Perhaps the simplest, yet still realistic, is to view benefits as the difference between the rates charged on a loan and that of the markets. Since the rates charged to borrow money are likely to differ from member to member, and the market rates charged are likely to differ as well, there will be a difference in the amount of benefits each member can potentially generate. Mathematically, the benefit for the i^{th} member from obtaining a loan at a credit union can be expressed as the difference between the market rate, r_{LMi}, and that of the credit unions for that member, r_{Li}, where $r_{MLi} - r_{Li} \geq 0$ for all $i = 1, 2, 3, \ldots, n$. Benefits for all n individuals in the membership base can be organized into the vector of size $1 \times n$, $\boldsymbol{b}$.

Total net benefits β generated by the credit union can be quantitatively measured by summing all the benefits of each individual member multiplied by the percentage of the total assets of the portfolio lent to that individual, that is

$$\beta = \boldsymbol{x}^t \boldsymbol{b}.$$

Due to the stochastic nature inherent in the operation of a depository institution, the rate of return on loans is uncertain ex ante any given period, and will only be known ex post. As a result of this, the rate credit unions initially charge on loans to any given member, r_{Li}, may differ significantly from the rate of return they receive on that loan, $\tilde{r}_i$. To account for this uncertainty in the model, we let the vector $\tilde{\boldsymbol{r}}$, represent all actual rates of return on loans for all $i = 1, 2, 3, \ldots, n$.

Although deposits could be incorporated into the model, their inclusion in this analysis is complicated by the way benefits are defined. For deposits to be included in the model, some form of a reserve requirement would also have to be included. Furthermore, to differentiate deposits from loans mathematically, deposits would have to be negative assets within the portfolio. Combined with the reserve requirement, the only feasible way to describe deposits is to set the total quantity of deposits equal to $1 - \frac{1}{q}$, where q is the ratio of deposits held in reserves. Because q is between zero and one, the total quantity of deposits will then be a negative number less than -1. This creates a dilemma when benefits generated from deposits are to be incorporated. Since the total sum of deposits is less than -1, the amount of benefits generated from one deposit, will be greater than the amount of benefits generated from one loan of the same size. This violates the assumption that credit unions are neutral in their preferences to borrowers and savers. In addition, since most of the analysis in the proceeding sections focus on how loan allocation changes and not necessarily deposits, the addition of deposits in the model is to some extent unwarranted. Therefore, for the sake of keeping the model somewhat simple, and not violating the assumptions of neutral preferences, deposits have been omitted.

Given that the actual return on the portfolio of loans will only be known ex post, these rates cannot be used. However, the expected rate of return for all assets can be estimated ex ante, provided the credit union knows the variance of the return on each loan. Using this, the expected rate of return on the portfolio of the credit union R_p can be described as

$$R_p = \boldsymbol{x}^t \bar{\boldsymbol{r}}$$

where $\bar{\boldsymbol{r}}$ denotes the vector of expected returns on all loans and deposits.

Because credit unions are non-profit financial intermediaries, they are prohibited by the non-distribution constraint from distributing earnings to owners and managers. As a result, credit unions will attempt to secure an expected rate of return that is equal to the costs associated with operating a credit union K. Therefore, $R_p \equiv K$. This condition also arises from the assumption that credit unions are neutral in their preferences to borrowers and savers. If a credit union favored savers to borrowers, then the credit union would attempt to allocate their portfolio such that $R_p > K$. This would allow the credit union to transfer excess revenue to depositors. Similarly, if a credit union favored borrowers to savers, then $R_p < K$, since credit unions would be attempting to lend at lower rates to generate a larger amount of benefits for members who borrower.

The random distribution of returns is given by the variance-covariance matrix Ω, where the covariance of the i^{th} and j^{th} loans is denoted σ_{ij}. The variance, or idiosyncratic risk, of the i^{th} loan is given by σ_i^2. It is assumed that that Ω is non-singular, which means that no loan can be represented as a linear combination of the other loans. As a result of the symmetry and non-singularity of the variance-covariance matrix, and since $\sigma_{ij} > 0$ for all i, j, Ω is positive definite. As discussed in Merton (1972), these conditions also imply that the inverse of the variance–covariance matrix Ω^{-1} will be positive definite as well. Provided this information, the total variance of a credit union's portfolio, σ_p^2, is given by the following quadratic equation

$$\sigma_p^2 = \boldsymbol{x}^t \Omega \boldsymbol{x}.$$

All possible combinations of portfolios can be constructed using the definitions for total net benefits accrued, the expected rate of return on the portfolio, and the variance of the portfolio. The frontier of portfolios which are efficient is the locus of portfolios that minimize the variance while maximizing the expected rate of return and benefits generated. The efficiency frontier will be the end product of solving the following quadratic programming problem

$$\min_{\{x\}} \frac{1}{2}\boldsymbol{x}^t \Omega \boldsymbol{x}$$

$$\begin{aligned} \textit{subject to}:\ \beta &= \boldsymbol{x}^t \boldsymbol{b} \\ K &= \boldsymbol{x}^t \bar{\boldsymbol{r}} \\ 1 &= \boldsymbol{x}^t \mathbf{1} \end{aligned} \qquad (1)$$

The variable of choice in this minimization problem is $\boldsymbol{x}$, since we are concerned with how the credit union will allocate its resources within the portfolio. To simplify the analytical derivation of the solution, $\boldsymbol{x}^t \Omega \boldsymbol{x}$ is multiplied by $\frac{1}{2}$. This will have no effect on the actual solution, and is a common technique used in the literature to simplify the analytics. To solve (1), we convert the problem to an unconstrained one by way of Lagragian multipliers. Doing so produces

$$\mathfrak{L} = \frac{1}{2}\boldsymbol{x}^t \Omega \boldsymbol{x} + \lambda_1(\beta - \boldsymbol{x}^t \boldsymbol{b}) + \lambda_2(K - \boldsymbol{x}^t \bar{\boldsymbol{r}}) + \lambda_3(1 - \boldsymbol{x}^t \mathbf{1})$$

The first order conditions for minimization are

$$\nabla \mathfrak{L} = \Omega \boldsymbol{x} - \lambda_1 \boldsymbol{b} - \lambda_2 \bar{\boldsymbol{r}} - \lambda_3 \mathbf{1} \equiv \mathbf{0}$$

These first order conditions can be manipulated algebraically to solve for the efficiency frontier

$$\boldsymbol{x} = \lambda_1 \Omega^{-1} \boldsymbol{b} + \lambda_2 \Omega^{-1} \bar{\boldsymbol{r}} + \lambda_3 \Omega^{-1} \mathbf{1}. \qquad (2)$$

Provided the assumption of positive definiteness of Ω^{-1} holds, the solution in (2) represents the locus of all portfolios that will minimize the variance of the portfolio and maximize the return and benefits accrued. To obtain a unique portfolio for the credit union, the utility function of the benefit maximizing credit union must be described.

The Utility Function of a Credit Union

For given values of β, R_p, and σ_p^2 the utility function of a credit union can be specified implicitly as

$$U = U(\beta, R_p, \sigma_p^2).$$

where U is assumed to be a continuously differentiable function in terms of β, R_p, and σ_p^2. Even though the utility function could be specified explicitly in terms of β, R_p, and σ_p^2, and various risk aversion coefficients could be incorporated in this manner, doing so, only adds additional complexity that is unwarranted for this analysis. The utility function is assumed to be additively separable in terms of σ_p^2, β, and R_p, which implies that cross-effects of the σ_p^2, β, and R_p are equal to zero. The justification for this is to simplify the mathematics that results in the proceeding comparative statics analysis.

The utility function is assumed to be strictly convex in β, that is

$$U_\beta > 0 \qquad U_{\beta\beta} < 0.$$ [1]

This is as one would expect given that the objective of credit unions is to maximize the benefits accrued to members. The traditional assumption of marginal diminishing utility is also assumed. This will help to guarantee that the benefit maximizing portfolio is unique.

---

[1] U_β is short hand notation denoting the first partial derivative of U with respect to β, $\frac{\partial U}{\partial \beta}$. Likewise, $U_{\beta\beta}$ is the second partial of U with respect to β, $\frac{\partial^2 U}{\partial \beta^2}$.

In view of the fact that credit unions are benefit maximizers, it seems somewhat counter intuitive that the rate of return on the portfolio of the credit union would have any direct effect on utility. However, due to the resources of credit unions being finite, and the rate of return being stochastic, credit unions will, ceteris paribus, prefer a higher rate of return on their portfolio since it will increase the likelihood that costs will be met for a given period. In the event that costs are not met, due to a lower rate of return, the credit union will be forced to require borrowers to pay a higher rate, and pay a lower rate on deposits, which will lead to decreased benefit production. Therefore, utility is increasing in R_p. As with β, the utility function is assumed to be strictly convex in R_p. Thus,

$$U_{R_p} > 0 \qquad U_{R_p R_p} < 0$$

Credit unions are assumed to be risk averse. Therefore, the utility function will consequently be decreasing in terms of the variance of the credit union's portfolio. Holding all else equal, higher variance implies less predictability, which can potentially lead to a shortage of funds. As argued above a shortage of funds will mean that costs will not be covered, which in turn results in less benefits being accrued to members. In addition, the normal axiom of marginal diminishing utility is assumed, but in reverse. That is, as more and more risk is taken on by the credit union, the disutility resulting from an addition unit of risk decreases. Mathematically, these relations can, as they are above, be expressed via the derivatives of the utility function, i.e.

$$U_{\sigma_p^2} < 0 \qquad U_{\sigma_p^2 \sigma_p^2} > 0.$$

To obtain the unique portfolio for a credit union, we must derive the marginal rates of substitution between benefits and portfolio variance$MRS_{\sigma_p^2,\beta}$ and the rate of return and the variance of the portfolio $MRS_{\sigma_p^2,R_p}$. The derivation of $MRS_{\sigma_p^2,\beta}$ and $MRS_{\sigma_p^2,R_p}$ follows from the implicit-function rule. Thus, given U,

$$MRS_{\sigma_p^2,\beta} = \left.\frac{d\sigma_p^2}{d\beta}\right|_{U=\overline{U},\, R_p=\overline{R_p}} \equiv -\frac{U_\beta}{U_{\sigma_p^2}}. \qquad (3)$$

Given the assumptions on the utility function, it can be concluded that $MRS_{\sigma_p^2,\beta} > 0$. To solve for $MRS_{\sigma_p^2,R_p}$ we again employ the implicit-function rule. The marginal rate of substitution between the risk and return of the portfolio is

$$MRS_{\sigma_p^2,R_p} = \left.\frac{d\sigma_p^2}{dR_p}\right|_{U=\overline{U},\beta=\overline{\beta}} \equiv -\frac{U_{R_p}}{U_{\sigma_p^2}}. \qquad (4)$$

Given the previous assumptions, $MRS_{\sigma_p^2,R_p} > 0$.

The Unique Portfolio of a Credit Union

Since the optimal portfolio that maximizes the utility of a credit union must be one that satisfies the marginal rates of substitution between benefits and variance, and return and variance. By the definition of Lagragian multipliers, it is possible to substitute (3) and (4) for the Lagragian multipliers in (2)

$$-\frac{U_\beta}{U_{\sigma_p^2}} = \lambda_1 \quad \text{and} \quad -\frac{U_{R_p}}{U_{\sigma_p^2}} = \lambda_2.$$

Substituting these values into (2) produces

$$\boldsymbol{x} = -\frac{U_\beta}{U_{\sigma_p^2}}\Omega^{-1}\boldsymbol{b} - \frac{U_{R_p}}{U_{\sigma_p^2}}\Omega^{-1}\bar{\boldsymbol{r}} + \lambda_3\Omega^{-1}\mathbf{1}. \qquad (5)$$

From (5), it is possible to solve for λ_3. Pre-multiplying both sides of (5), by $\mathbf{1}^t$ yields

$$\mathbf{1}^t\boldsymbol{x} = -\frac{U_\beta}{U_{\sigma_p^2}}\mathbf{1}^t\Omega^{-1}\boldsymbol{b} - \frac{U_{R_p}}{U_{\sigma_p^2}}\mathbf{1}^t\Omega^{-1}\bar{\boldsymbol{r}} + \lambda_3\mathbf{1}^t\Omega^{-1}\mathbf{1},$$

which can be manipulated algebraically to solve for λ_3. Thus,

$$\lambda_3 = \frac{1}{C}\left[1 + \frac{U_\beta}{U_{\sigma_p^2}}A + \frac{U_{R_p}}{U_{\sigma_p^2}}B\right]$$

Where:

$$A = \mathbf{1}^t\Omega^{-1}\mathbf{b} \qquad B = \mathbf{1}^t\Omega^{-1}\mathbf{r} \qquad C = \mathbf{1}^t\Omega^{-1}\mathbf{1}$$

Substituting in the solution to λ_3 into (5), produces

$$\boldsymbol{x}^* = -\frac{U_\beta}{U_{\sigma_p^2}}\Omega^{-1}\boldsymbol{b} - \frac{U_{R_p}}{U_{\sigma_p^2}}\Omega^{-1}\bar{\boldsymbol{r}} + \frac{1}{C}\left[1 + \frac{U_\beta}{U_{\sigma_p^2}}A + \frac{U_{R_p}}{U_{\sigma_p^2}}B\right]\Omega^{-1}\mathbf{1}, \qquad (7)$$

which is the solution to the unique portfolio for the benefit maximizing credit union.

Probability of Credit Union Failure

Given the solution to the unique portfolio, we can now analyze how various changes in the parameters of this model will increase or decrease the probability of credit union failure. Credit union failure will occur in the event that costs significantly exceed revenue in a given period. As a result of the stochastic environment in which credit unions operate, it is impossible to determine precisely when this event may occur. The best that can be done is to estimate the likelihood that failure will occur in a single period.

For the purpose of this analysis, the likelihood of failure will be said to increase if the probability that revenue will be below costs for a given period increases. To obtain this relationship, one would have to either know the exact distribution of the returns on a portfolio, or assume a distribution from basic assumptions. Given that the acquisition of the true distribution of returns on a credit union's portfolio is somewhat unfeasible, but that the mean R_p and variance σ_p^2 are assumed to be known, an appeal to Chebyshev's inequality can be made in order to estimate the upper bound of the probability of failure.

Following Blair and Heggestad, Koehn and Santomero, and Roy, provided the characteristics of the credit union's unique portfolio, and letting $\tilde{R}_p$ be a random variable denoting the actual return on the portfolio, Chebyshev's inequality states

$$P\{\tilde{R}_p - R_p > k\sigma_p^2\} \leq \frac{1}{k^2}. \ (8)$$

By letting $-1 = R_p - k\sigma_p^2$, implies $k = \frac{R_p+1}{\sigma_p}$ Thus, (8) can be re-written as

$$P\{\tilde{R}_p < -1\} \leq \frac{\sigma_p^2}{(R_p + 1)^2} = P. \quad (9)$$

From (9), it can be inferred that an increase in the probability of failure will occur if, ceteris paribus, σ_p^2 increases, or, ceteris paribus, R_p decreases.

The upper bound of the probability of failure can be depicted in mean-variance space as the square of the reciprocal of the slope of the ray α, where the ray's intercept is -1. This is depicted along with the efficiency frontier and highest indifference curve in Figure 1. Graphically, the less step α, the higher the probability of failure is.

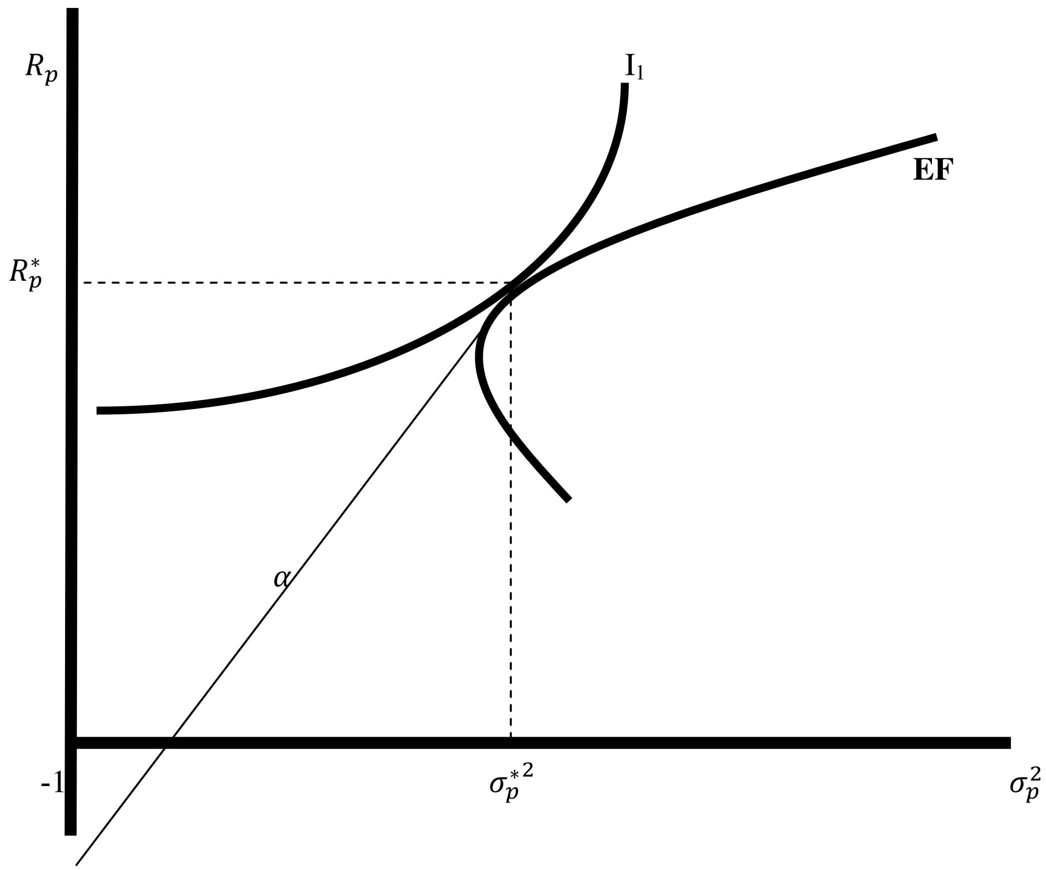

Figure 1. Unique portfolio in mean-variance space. The unique portfolio will be the portfolio in which the highest attainable indifference curve is tangent to the efficiency frontier.

Since the purpose of the analysis will be to see how various changes in the parameters change the mean and variance of the portfolio, it is first necessary to solve (7) for the variance of the portfolio σ_p^2 and the expected rate of return of the portfolio R_p. Using the solution to the unique portfolio (7) and multiplying it by $\boldsymbol{x}^t\Omega$, results in

$$\boldsymbol{x}^t\Omega\boldsymbol{x} = -\frac{U_\beta}{U_{\sigma_p^2}}\boldsymbol{x}^t\Omega\Omega^{-1}\boldsymbol{b} - \frac{U_{R_p}}{U_{\sigma_p^2}}\boldsymbol{x}^t\Omega\Omega^{-1}\bar{\boldsymbol{r}} + \frac{1}{C}\left[1 + \frac{U_\beta}{U_{\sigma_p^2}}A + \frac{U_{R_p}}{U_{\sigma_p^2}}B\right]\boldsymbol{x}^t\Omega\Omega^{-1}\mathbf{1}$$

which simplifies to

$$\sigma_p^2 = -\frac{U_\beta}{U_{\sigma_p^2}}\beta - \frac{U_{R_p}}{U_{\sigma_p^2}}K + \frac{1}{C}\left[1 + \frac{U_\beta}{U_{\sigma_p^2}}A + \frac{U_{R_p}}{U_{\sigma_p^2}}B\right]. \quad (10)$$

Following the same process to solve for R_p, requires multiplying (7) by $\boldsymbol{r}^t$, which produces

$$\boldsymbol{r}^t\boldsymbol{x} = -\frac{U_\beta}{U_{\sigma_p^2}}\boldsymbol{r}^t\Omega^{-1}\boldsymbol{b} - \frac{U_{R_p}}{U_{\sigma_p^2}}\boldsymbol{r}^t\Omega^{-1}\bar{\boldsymbol{r}} + \frac{1}{C}\left[1 + \frac{U_\beta}{U_{\sigma_p^2}}A + \frac{U_{R_p}}{U_{\sigma_p^2}}B\right]\boldsymbol{r}^t\Omega^{-1}\mathbf{1}$$

and simplifies to

$$R_p = -\frac{U_\beta}{U_{\sigma_p^2}}\boldsymbol{r}^t\Omega^{-1}\boldsymbol{b} - \frac{U_{R_p}}{U_{\sigma_p^2}}\boldsymbol{r}^t\Omega^{-1}\bar{\boldsymbol{r}} + \frac{1}{C}\left[1 + \frac{U_\beta}{U_{\sigma_p^2}}A + \frac{U_{R_p}}{U_{\sigma_p^2}}B\right]\boldsymbol{r}^t\Omega^{-1}\mathbf{1}. \quad (11)$$

Equations (10) and (11) tell us what the mean and variance of the credit union's unique portfolio will be for a given set of parameters. Using (9), (10) and (11), and Figure 1 it is possible to acquire what effects non-interest costs, systematic risk, and competition will have on the probability of failure via comparative static analysis.

Effect of Non-Interest Costs on the Probability of Failure

Intuitively, one would expect that higher costs will increase the probability of failure, since higher costs require that a credit union must garnish a higher rate of return on its portfolio. One of the most basic tenets of finance states that a higher rate of return can only come with the addition of more risk. Thus, by the definition in this thesis used to define the probability of failure, it seems inevitable that an increase in non-interest costs K, will increase the probability of failure. However, as will be shown, this does not necessarily have to be the end result.

The two comparative static terms of interest for this analysis are $\frac{\partial \sigma_p^2}{\partial K}$ and $\frac{\partial R_p}{\partial K}$. Signing these terms will make it possible to determine whether the upper bound of the probability of failure will increase or decrease as a result of an increase in non-interest costs. Taking the derivative of (10) with respect to costs yields,

$$\frac{\partial \sigma_p^2}{\partial K} = -\frac{U_{R_p}}{U_{\sigma_p^2}}.$$

Since, $-\frac{U_{R_p}}{U_{\sigma_p^2}} > 0$, it follows that $\frac{\partial \sigma_p^2}{\partial K} > 0$. Therefore, an increase in non-interest costs will result in the variance of the credit union's portfolio increasing. Since, $R_p \equiv K$ as a result of the non-distribution constraint, $\frac{\partial R_p}{\partial K} = 1$. These results are as would be expected, since the non-distribution constraint will force the credit union to seek a higher rate of return in the event that costs increase, and as a result of the credit union seeking a higher rate of return the variance necessarily must increase as well.

With reference to Chebyshev's Inequality, since both R_p and σ_p^2 are positively related to costs, it cannot be directly determined as to whether or not the probability of failure will increase or not, as this will depend on the magnitudes of the two derivatives. Nonetheless, with the information provided it is possible to describe what circumstances would give rise to $\frac{\partial \sigma_p^2}{\partial K} > \frac{\partial R_p}{\partial K}$ and $\frac{\partial R_p}{\partial K} > \frac{\partial \sigma_p^2}{\partial K}$. Since $\frac{\partial R_p}{\partial K} = 1$ and $\frac{\partial \sigma_p^2}{\partial K} = -\frac{U_{R_p}}{U_{\sigma_p^2}}$, in the event that $-\frac{U_{R_p}}{U_{\sigma_p^2}} > 1$, that is the marginal rate of substitution between risk and return in greater than 1, the probability of failure will increase. Manipulating the terms in $-\frac{U_{R_p}}{U_{\sigma_p^2}} > 1$ and taking absolute values, it is apparent that this will only occur if $\left|U_{R_p}\right| > \left|U_{\sigma_p^2}\right|$, that is the marginal utility gained from an additional unit of return is greater than the utility lost from an additional unit of risk. Conversely, if $\left|U_{R_p}\right| < \left|U_{\sigma_p^2}\right|$, then the probability of failure will decline, since the utility lost from taking on addition unit of risk is greater the benefit gained from a higher rate return. This will serve to limit the amount of risk the credit union is willing to take on, thus resulting in a smaller increase in σ_p^2.

Effects of Increased Systematic Risk on Credit Union Failure

The purpose of this section is to analyze the effects of changes in the systematic risk of a credit union's portfolio on the probability of failure. More specifically, this section seeks to derive whether credit unions operating under multiple common bonds have lower probabilities of failure than that of credit unions operating under a single common bond. Credit unions that operate under a single bond are restricted in their membership bases to specific residential areas, specific employers, or a specific association. Because of this, credit unions are held hostage to the economic success of their membership base. This implies that the covariance between all individuals in the membership base will be higher than that of a credit union operating under multiple common bonds. Therefore, it is expected that credit unions operating under the single common bond will have a higher probability of failure.

To ascertain the effects of the single common bond on the likelihood of failure, we will need to determine what effect an increase in the covariance of all members has on Chebyshev's Inequality. Thus, the comparative static terms of interest are

$$\frac{\partial \sigma_p^2}{\partial \Omega} \text{ and } \frac{\partial R_p}{\partial \Omega}.$$

In order to ascertain the effect that increased homogeneity has on the membership base, the derivative of σ_p^2 and R_p with respect to the covariance matrix would have to be computed tediously via matrix calculus. So, in order to avoid these tedious analytics, qualitative techniques will be used.

Graphically, the portfolio of a credit union operating under a single common bond can be seen in Figure 2, where it is labeled *SB*. The portfolio for a credit union operating under multiple common bonds with a membership base of the same size is also depicted in Figure 2 and is labeled *MB*. As it can be inferred from Figure 2, the shape of the efficiency frontier for the single common bond credit union is still quadratic, but the curvature has decreased. This is the result of the increased systematic risk in the portfolio for the single common bond credit union. The current depiction shows an increase in the probability of failure for credit unions operating under the single common bond, since the ray extending from -1 and intersecting the point where the unique portfolio is located is flatter for the single bond credit union then the ray for the multiple bonds credit union.

The reason for the outcome in Figure 2 can be attributed to the relative risk aversion of the utility function of credit unions. In this case, as a result of more risk, credit unions attempt to minimize the effects of increased variance, by accepting a lower expected rate of return on assets. However, because all the portfolios on the efficiency frontier for the single common bond credit union have higher variances than that of the multiple common bonds credit union, the variance of the portfolio must also increase. Therefore, since the expected rate of return is decreasing and the variance of the portfolio is increasing, the upper bound on the probability of failure increases for the single common bond credit union.

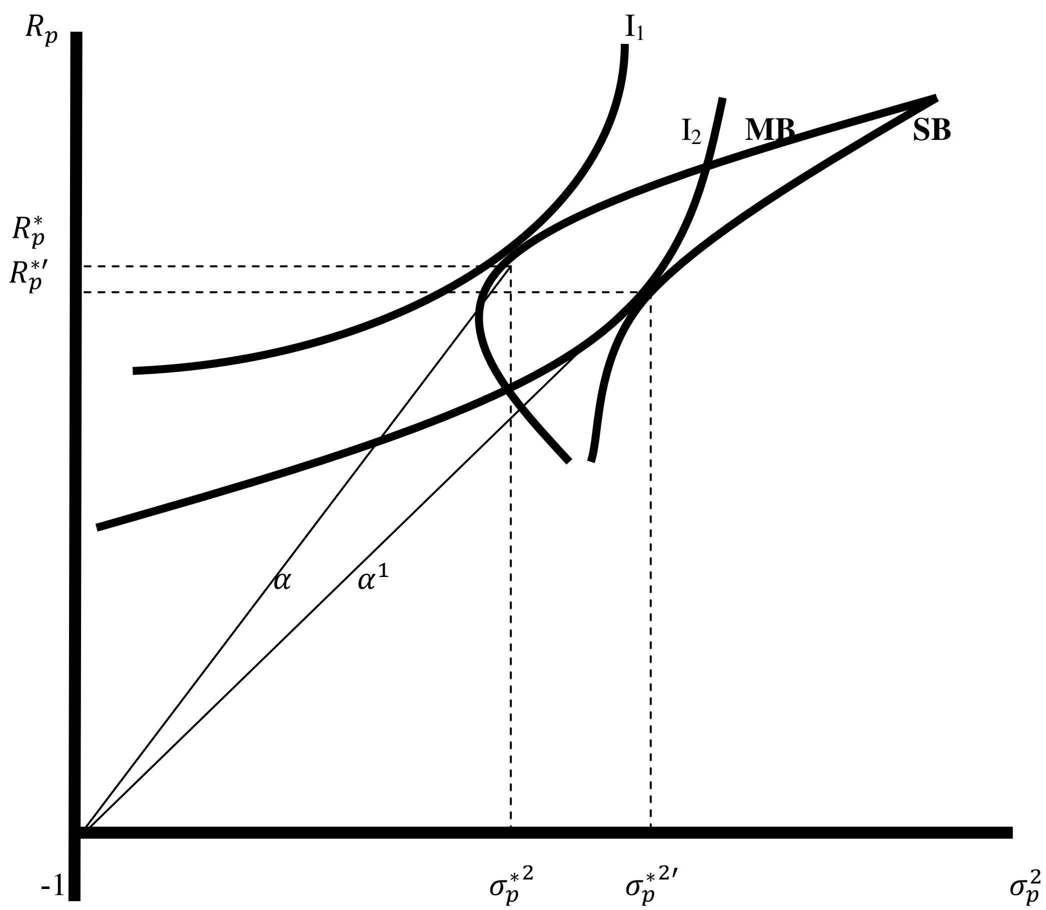

Figure 2. Unique portfolios for multiple and single common bond credit unions. In this figure the expected rate of return is lower, and the variance is higher for the single common bond credit union. This implies the probability of failure is higher.

The second possible case is an increase in both the expected return on the portfolio and the variance of the portfolio. This scenario is depicted in Figure 3. Although the expected return has increased, the increase in the magnitude of the variance is greater. Therefore, the likelihood of failure has increased. Graphically, this information can be inferred from the differences in the slopes of the rays, α' and α. Since the slope of α' is less steep than the slope of α, the probability of failure is higher for the single bond credit union. Additionally, from Figure 3, it can be seen that the difference in the variance for the single bond credit union's unique portfolio, $\sigma_p^{*2\prime}$, and that of the multiple bond, σ_p^{*2}, is larger than the difference in the expected rate of return for the single common bond credit union, $R_p^{*\prime}$, and that of the multiple, R_p^{*}. This scenario will occur in the event that the utility function for credit unions is relatively less risk averse than the utility function described in the first scenario.

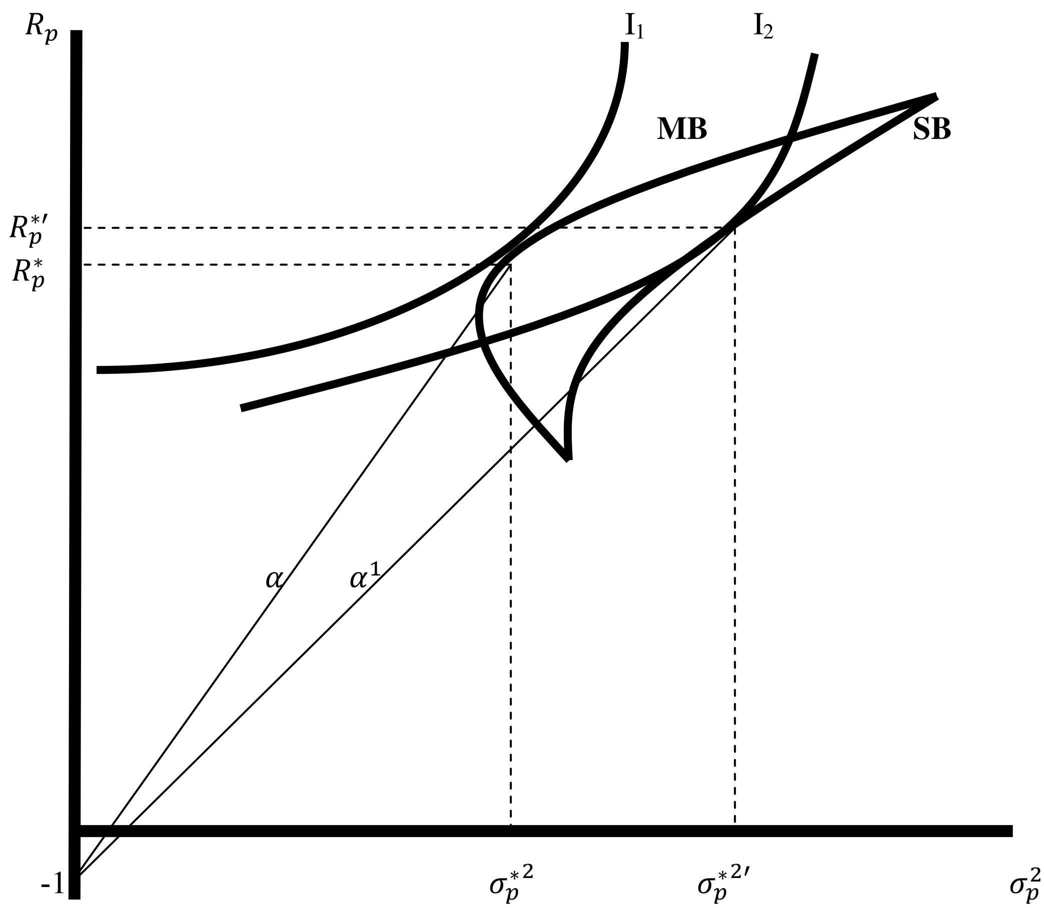

Figure 3. Unique portfolios with an increase in expected return for single common bond credit union. In this figure the single common bond credit union's unique portfolio has a higher expected rate of return. However, the variance on that portfolio is significantly larger than that of the multiple common bonds credit union.

Assuming that all credit unions have identical utility functions, it would be impossible for a single common bond credit union to have a higher probability of failure than a multiple common bonds credit union. The reason from this is that the efficiency frontier of single common bond credit unions will always contain portfolios that for the same expected return as a multiple bonds credit unions have a higher variance. Therefore, if the utility functions are the same, which implies they both have the same relative risk aversion, single common bond credit unions will always allocate their portfolios such that the difference in the variance between their portfolio and a multiple bonds one is greater than the difference in the expected rate of return.

If the utility functions are not the same, then there is a chance that the probability of failure may decrease for the single common bond credit union. For this event to occur it must be the case that the multiple common bonds credit union is significantly less risk averse than that of a single common bond credit union. By being less risk averse, the multiple bonds credit union will hold a portfolio with a large amount of variance and a high expected rate of return. The single bond credit union, on the other hand, will hold a portfolio that has a much lower rate of return and lower variance, which is the byproduct of being relatively more risk averse. As it can be seen in Figure 4, because the differences in variances are greater than the differences in the expected rate of return, the probability of failure for the multiple bonds credit union will be higher.

However, for this event to occur, not only does the multiple bonds credit union have to be considerably less risk than the single bond credit union, but the differences in systematic risk must also be somewhat minimal. If the differences are significantly large, then it would be impossible for a single common bond credit union to hold a portfolio that decreased its probability of failure relative to that of a multiple common bonds credit union. To prove this, note that if the efficiency frontier for the single common bond credit union had higher systematic risk than the one depicted in Figure 4, all portfolios along it would be to the right of the ray α. Thus, there would not exist an attainable portfolio for the single common bond credit union that would allow it to have a lower probability of failure than that of the multiple common bonds credit union.

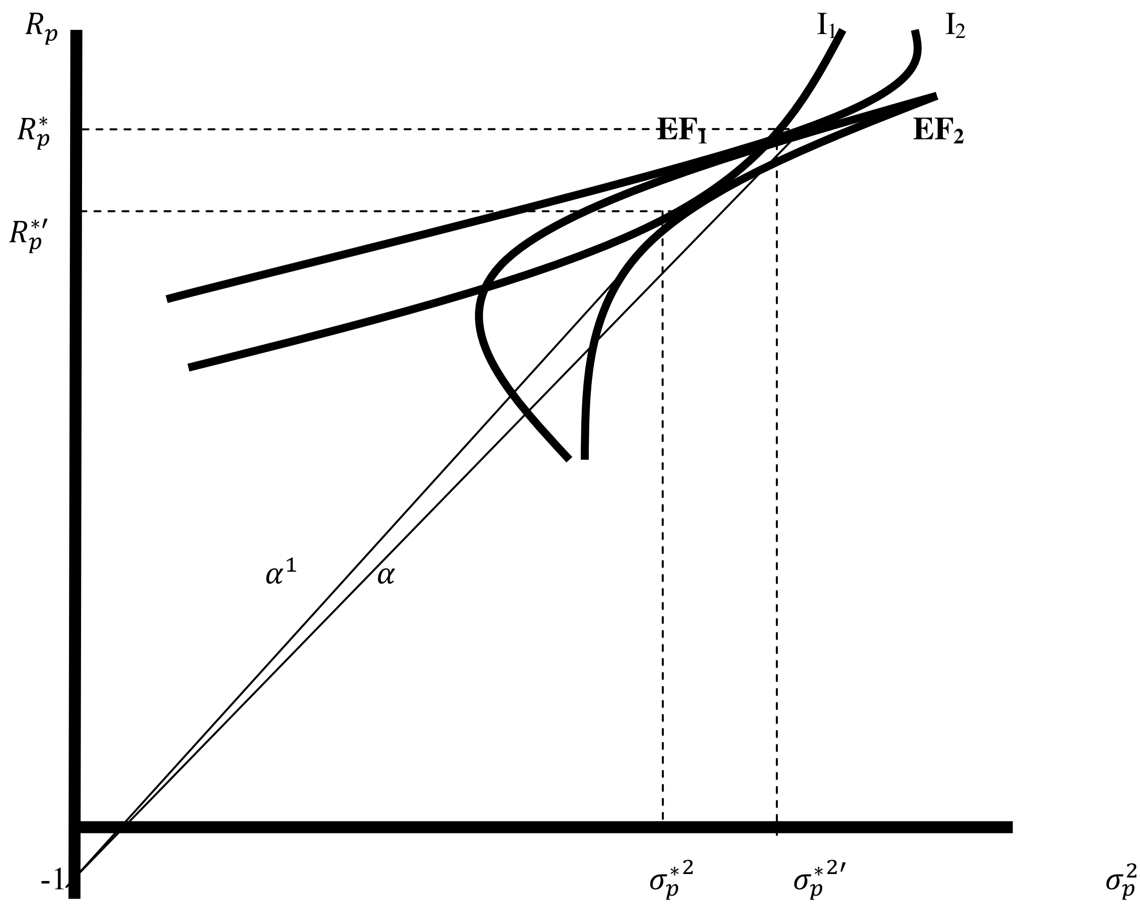

Figure 4. A decrease in the probability of failure for a single bond credit union. This figure illustrates the only scenario in which the probability of failure will decrease for the single common bond credit union.

Although differences in the utility functions for credit unions can lead to a decrease in the probability of failure, given the number of conditions that must be satisfied for this event to occur, it seems somewhat improbable. Furthermore, it is important to note that differences in the utility functions between multiple common bonds credit unions and single bond credit unions will not necessarily imply that the probability of failure will increase. In fact if single common bond credit unions are significantly more risk averse, or multiple common bonds credit unions are slightly less risk averse, then probability of failure will be higher for single common bond credit unions.

Effect of Increased Competition on the Probability of Credit Union Failure

As a result of the growth of the credit union industry over the past fifty years, the competition between credit unions and other depository institutions has increased significantly. Due to the way benefits are defined in this analysis, the direct effect of increased competition on credit unions will be decreased benefits $\boldsymbol{b}$. The justification for this is that more competition should lower the market rate for loans, and since benefits are defined as the difference between the market rate and that of the credit union's rate on loans, a lower market rate will correspond to lower benefits, all else equal. Thus, in order to determine what effect increased competition has on the probability of credit union failure, we must determine the sign of $\frac{\partial \sigma_p^2}{\partial \boldsymbol{b}}$ and $\frac{\partial R_p}{\partial \boldsymbol{b}}$.

How σ_p^2 changes with respect to a change in the vector $\boldsymbol{b}$ is determined by the direction of the gradient of σ_p^2 with respect to the vector $\boldsymbol{b}$, thus

$$\frac{\partial \sigma_p^2}{\partial \boldsymbol{b}} = -\frac{U_\beta}{U_{\sigma_p^2}}\boldsymbol{x} + \frac{U_\beta}{U_{\sigma_p^2}}\frac{1}{C}\Omega^{-1}\mathbf{1} \qquad (11)$$

Since $-\frac{U_\beta}{U_{\sigma_p^2}}\boldsymbol{x}$ is non-negative, while $\frac{U_\beta}{U_{\sigma_p^2}}\frac{1}{C}\Omega^{-1}\mathbf{1}$ is non-positive, it is impossible to conclusively sign the direction of $\frac{\partial\sigma_p^2}{\partial\boldsymbol{b}}$ with the given information. However, it is possible to determine the circumstances that will cause $\frac{\partial\sigma_p^2}{\partial\boldsymbol{b}}$ to be positive or negative. By manipulating (11) algebraically,

$$\frac{\partial\sigma_p^2}{\partial\boldsymbol{b}} = -\frac{U_\beta}{U_{\sigma_p^2}}\left(\boldsymbol{x} - \frac{1}{C}\Omega^{-1}\mathbf{1}\right).$$

Thus, it is now possible to see that in order to guarantee $\frac{\partial\sigma_p^2}{\partial\boldsymbol{b}} > \mathbf{0}$, the magnitude of the portfolio, $\boldsymbol{x}$, must be greater than the magnitude of $\frac{1}{C}\Omega^{-1}\mathbf{1}$. Unfortunately, the complexity of $\frac{1}{C}\Omega^{-1}\mathbf{1}$ makes it difficult to use any economic intuition to determine when this case will occur or not.

The sign of $\frac{\partial R_p}{\partial\boldsymbol{b}}$ can be obtained in a similar fashion, where the gradient of R_p is

$$\frac{\partial R_p}{\partial\boldsymbol{b}} = -\frac{U_\beta}{U_{\sigma_p^2}}\Omega^{-1}\boldsymbol{r} + \frac{U_\beta}{U_{\sigma_p^2}}\frac{B}{C}\Omega^{-1}\mathbf{1}. \qquad (12)$$

As with $\frac{\partial\sigma_p^2}{\partial\boldsymbol{b}}$, $\frac{\partial R_p}{\partial\boldsymbol{b}}$ cannot be signed conclusively with the information provided by (12), since

$-\frac{U_\beta}{U_{\sigma_p^2}}\Omega^{-1}\boldsymbol{r} \geq 0$ while

$\frac{U_\beta}{U_{\sigma_p^2}}\frac{B}{C}\Omega^{-1}\mathbf{1} \leq \mathbf{0}$.

To better see what conditions will guarantee $\frac{\partial R_p}{\partial\boldsymbol{b}} > \mathbf{0}$, a similar process can be followed. Manipulating (11) to better understand what situations will give rise to certain signs, yields

$$\frac{\partial R_p}{\partial \boldsymbol{b}} = -\frac{U_\beta}{U_{\sigma_p^2}}\Omega^{-\mathbf{1}}\left(\boldsymbol{r} - \frac{B}{C}\mathbf{1}\right).$$

Because $-\frac{U_\beta}{U_{\sigma_p^2}}\Omega^{-\mathbf{1}} \geq \mathbf{0}_{n,n}$[2], the sign of the direction of $\frac{\partial R_p}{\partial \boldsymbol{b}}$ will be dependent on whether $|\boldsymbol{r}| \lesseqqgtr \left|\frac{B}{C}\mathbf{1}\right|$. As with $\frac{\partial \sigma_p^2}{\partial \boldsymbol{b}}$, the conditions needed to guarantee $\frac{\partial R_p}{\partial \boldsymbol{b}} > \mathbf{0}$ are too complex to analyze using any economic intuition.

Unfortunately, due to the complexity of $\frac{\partial \sigma_p^2}{\partial \boldsymbol{b}}$ and $\frac{\partial R_p}{\partial \boldsymbol{b}}$ I am prohibited from using specific economic cases to conclusively determine what conditions will increase or decrease the probability of failure for a credit union in the face of increased competition. However, given that the conditions for when $\frac{\partial \sigma_p^2}{\partial \boldsymbol{b}}$ is strictly positive and $\frac{\partial R_p}{\partial \boldsymbol{b}}$ is strictly negative are known, it is possible to discuss what scenarios will lead to a higher probability of failure. Since increased competition will cause a decrease in $\boldsymbol{b}$, the probability of failure will increase if $\frac{\partial \sigma_p^2}{\partial \boldsymbol{b}} < \mathbf{0}$ and $\frac{\partial R_p}{\partial \boldsymbol{b}} > \mathbf{0}$. As it can be seen from above, this occurs only when $\left|\frac{1}{C}\Omega^{-1}\mathbf{1}\right| > |\boldsymbol{x}|$, and $|\boldsymbol{r}| > \left|\frac{B}{C}\mathbf{1}\right|$. In addition, the probability of failure will also increase in the case that $\frac{\partial \sigma_p^2}{\partial \boldsymbol{b}} < \mathbf{0}$ and $\frac{\partial R_p}{\partial \boldsymbol{b}} < \mathbf{0}$, if $\left|\frac{\partial \sigma_p^2}{\partial \boldsymbol{b}}\right| > \left|\frac{\partial R_p}{\partial \boldsymbol{b}}\right|$. For this to occur, it must be the case that $|\boldsymbol{r}| < \left|\frac{B}{C}\mathbf{1}\right|$, which guarantees $\frac{\partial R_p}{\partial \boldsymbol{b}} < \mathbf{0}$.

Conclusion

The theory developed here presents a model to describe the optimal unique portfolio allocation for a benefit maximizing credit union by altering the existing framework of modern portfolio theory to include benefits produced by the credit union, which are a key factor in the

[2] $\mathbf{0}_{n,n}$ denotes a matrix of size $n \times n$ with zeros in every entry.

credit union decision making process. Through the specification of a utility function in terms of benefits β, expected rate of return on the portfolio R_p, and variance of the portfolio σ_p^2, and the derivation of an efficiency frontier that included the benefits generated by loans, the unique portfolio for an arbitrary credit union was obtained. Using the derivation of the unique portfolio, a comparative static analysis was performed in order to ascertain what characteristics pertaining to specific credit unions increase or decrease the probability of failure. This analysis used Chebyshev's Inequality in order to determine the upper bound of the probability of failure. Using Chebyshev's Inequality the effects of increased costs, increased systematic risk, and increased competition on the probability of failure were analyzed. For increased costs, it was found that whether or not the probability of failure increased as costs increased was dependent on whether the marginal utility with respect to the rate of return on the portfolio was greater than the utility lost from taking on one more unit of risk. With changes in systematic risk, it was determined through qualitative analysis that the probability of failure would always be higher for credit unions operating under a single common bond, unless multiple bonds credit unions were less risk averse than single common bond credit unions, in which case the probability of failure would increase. The effects of increased competition on the probability of failure could not be directly ascertained using economic theory, since the complexity of solutions were too great for any interpretation to be had from them. Nonetheless, the solutions did allow for some conclusions to be drawn about when the probability of failure would be greater.

The model used in this discussion is, for the most part, unlike any model previously used in the literature to describe credit union behavior. As a result of this, there may be potential flaws in its design, which may have led to incorrect conclusions. Additionally, the complexity of solutions, which is not entirely surprising given the addition of benefits in the model, were difficult to interpret from the stand point of economic inquiry. Ideally, future models that used portfolio theory to describe credit union behavior would be able to find ways to alter the model in order to obtain solutions that are easier to interpret. Also, future models should be able to analytically determine the effects of changes in systematic risk and size of the credit union on the location of the unique portfolio. Finally, a major issue with this model is that adding more constraints to the model, in order to add more realism or more parameters in which comparative statics can be performed, adds an exponentially increasing amount of complexity to the solutions of the unique portfolio. This serves to increase the difficulty in interpreting solutions.

In conclusion, most of questions raised within this thesis cannot be answered via theoretical arguments, since the dependency on changes in the unique portfolio will be on the magnitudes of various parameters. Thus, the resolution of these questions becomes an empirical issue. Nonetheless, this thesis presents a new model describing credit union behavior from the perspective of portfolio theory as a benefit maximizer, in which the effects of various environmental and individual characteristics can be analyzed to see how they affect the probability of credit union failure.

References

Blair, R.D., & Heggestad, A. A. (1978). Bank Portfolio Regulation and the Probability of Bank Failure. *Journal of Money, Credit, and Banking,* 10(1), 88-93.

Brown, R. & O'Connor, Ian. (1999). Efficiency, Bonds of Association and Exit Patterns in Credit Unions. *Annals of Public and Cooperative Economics,* 70(1), 5-23.

Crapp, H. R. & Stevenson, M. S. (1987). Development of a Method to Assess the Relevant Variables and the Probability of Financial Distress. *Australian Journal of Management,* 12(2), 221-236.

Hansmann, H. (1987). Economic Theories of the Non-Profit Sector. In W. Powell, (Ed.), *The Nonprofit Sector.* (27-42). New Haven, CT: Yale University Press.

Hart, O. D. & Jaffree, D. M. (1972). On the Application of Portfolio Theory to Depository Financial Institutions. *Review of Economic Studies,* 41, 129-147.

Kim, D. & Santomero, A. M. (1988). Risk in Banking and Capital Regulation. *The Journal of Finance,* 43(5), 1219-1233

Klein, M. A. (1971). A Theory of the Banking Firm. *Journal of Money, Credit, and Banking,* 3(2), 205-218.

Koehn, M. & Santomero, A. M. (1980). Regulation of Bank Capital and Portfolio Risk. *The Journal of Finance,* 35(5), 1235-1244.

Markowitz, H. M. (1952). Portfolio Selection. *The Journal of Finance,* 7(1), 77-91.

Merton, R. C. (1972). An Analytic Derivation of the Efficient Portfolio Frontier. *The Journal of*

Financial and Quantitative Analysis, *7*(4), 1851-1874.

Rahmen, M. & McNeil, D. W. (1999). Portfolio Behavior of Large U.S. Credit Unions under Uncertainty: A Theoretical Exposition. *New York Economic Review,* Fall, 23-29.

Sealey, Jr., C. W. (1980). Deposit Rate-Setting , Risk Aversion, and the Theory of Depository Financial Intermediation. *The Journal of Finance,* 35(5), 1139-1154.

Smith, D. J., Cargill, T. F. & Meyer, R. A. (1980). An Economic Theory of a Credit Union. *The Journal of Finance,* 38(2), 519-528.

Smith, D. J. (1984). A Theoretical Framework for the Analysis of Credit Union Decision Making. *The Journal of Finance,* 38(2), 519-528.

Smith, D. J. (1984). Credit Union Rate and Earnings Retention Decisions under Uncertainty and Taxation. *Journal of Money, Credit, and Banking,* 20(1), 119-131.

Walter, J. R. (2006). Not your Father's Credit Union. *Economic Quarterly,* 92(4), 353-277.