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Scattering of a Point-Like Anapole

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Purpose

The purpose of this research is to investigate the electromagnetic interactions between a Majorana particle and an electron in the framework of quantum electrodynamics.

Background

Scattering is a physical process where the trajectories of incident and target particles are altered by an interaction between the particles. The majority of the information we know about subatomic particles is obtained via scattering processes.

Majorana fermions are particles of half-integer spin which are their own antiparticles. As a result, the only static electromagnetic property is the anapole moment.

Scattering

- Incident/target particles interact via some force
- Changes in trajectory depend on the energy of the particles and results are expressed in terms of the scattering angle θ

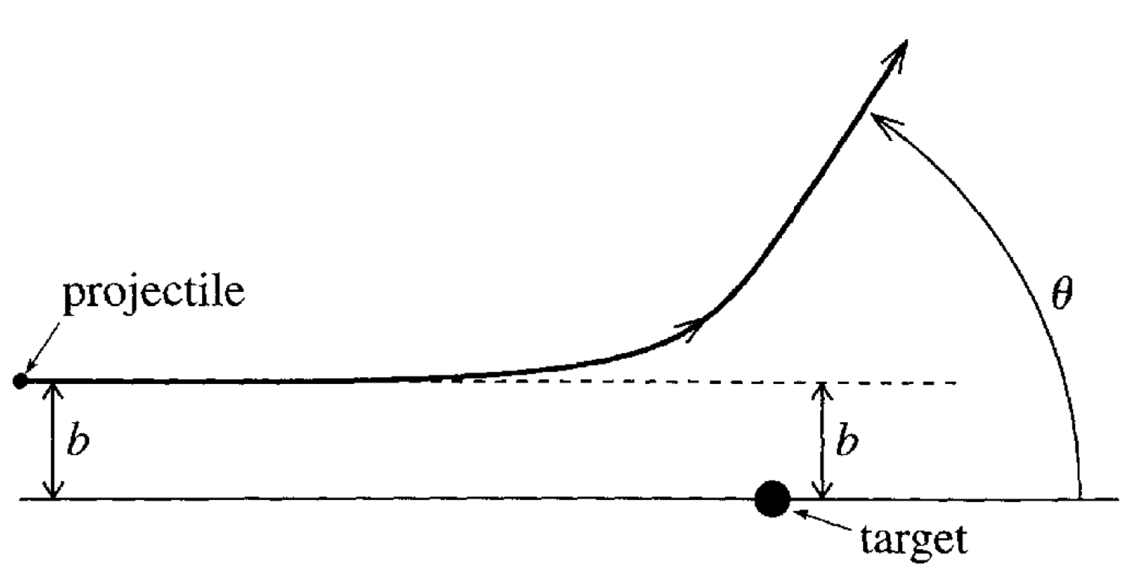


Figure 1. J. R. Taylor. Classical Mechanics. Sausalito, Calif: University Science Books, 2005. 557-86. Print.

Example: Rutherford scattering describes the deflection of a non-relativistic ion beam (charge $Z_2 e$, mass m, and speed v_0) and a massive nucleus (charge $Z_2 e$) at rest. The differential scattering cross section is

$$\frac{d\sigma}{d\Omega} = \left(\frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 m v_0^2}\right)^2 \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

Rutherford scattering is the phenomenon that led to the discovery of the atomic nucleus – incident alpha particles are fired at a gold foil and interact with the gold nuclei via electrostatic forces. Rutherford scattering has a characteristic angular dependence of $(\sin\theta)^{-4}$.

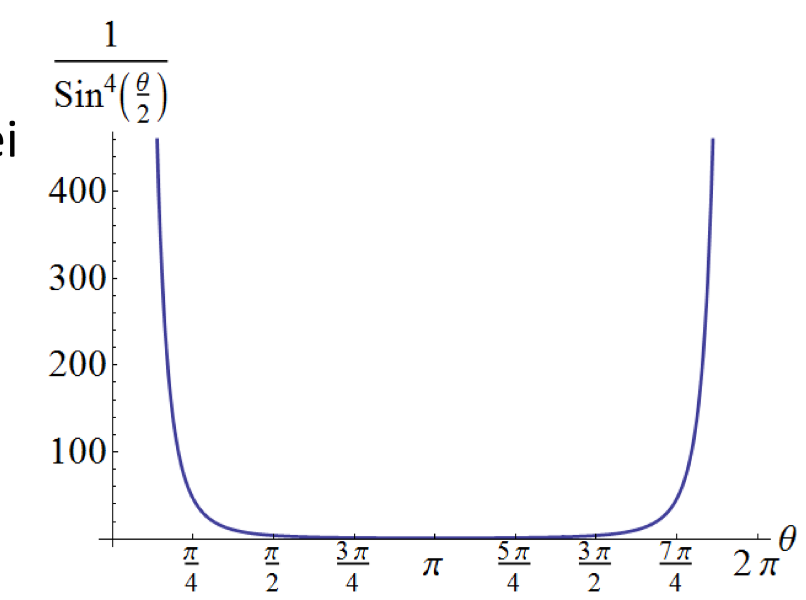


Figure 2

Neutral Targets

Scattering can occur with electrically neutral targets as well. The three simplest classes of these neutral systems are electric dipoles, magnetic dipoles, and anapoles.

Majorana Particles

Majorana particles interact via the anapole moment which can be computed from quantum field theory. The interaction vertex is

$$f_A\left(q^2\right)\left(q^2\gamma^\mu - q^\mu q_\lambda \gamma^\lambda\right)\gamma^5$$

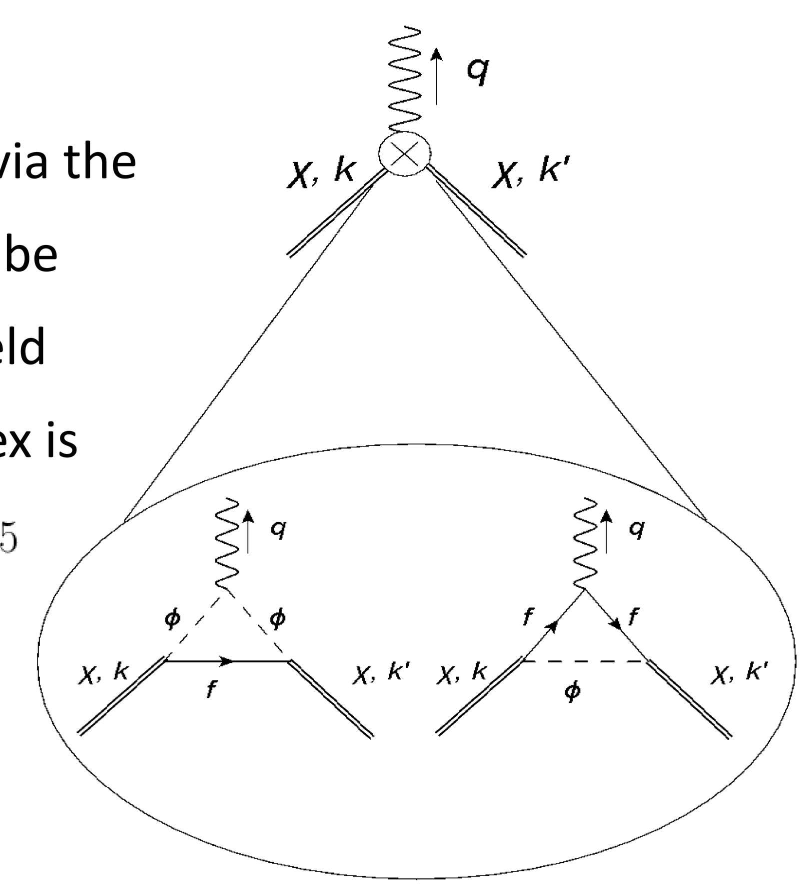


Figure 3

Feynman Diagram for Anapole-Electron Scattering

An electron (e, momentum *p*) interacts with a Majorana particle (χ, momentum k). Their trajectories are altered to the new momenta *p′* and *k′*. Each object in the diagram (e.g. particle lines and vertices) has a corresponding mathematical term which are combined to form an equation for the amplitude of the interaction.

$$-i\mathcal{M} = \left[\overline{u}(p')(ig_e\gamma^\mu)u(p)\right]\frac{-ig_{\mu\nu}}{q^2} \cdot \left[\overline{u}(k')f_A\left(q^2\right)\left(q^2\gamma^\nu - q^\nu q_\lambda\gamma^\lambda\right)\gamma^5 u(k)\right]$$

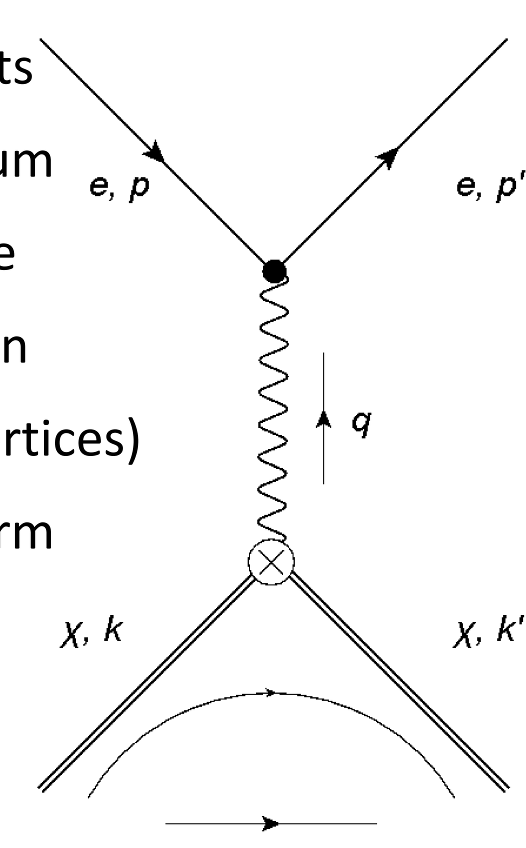


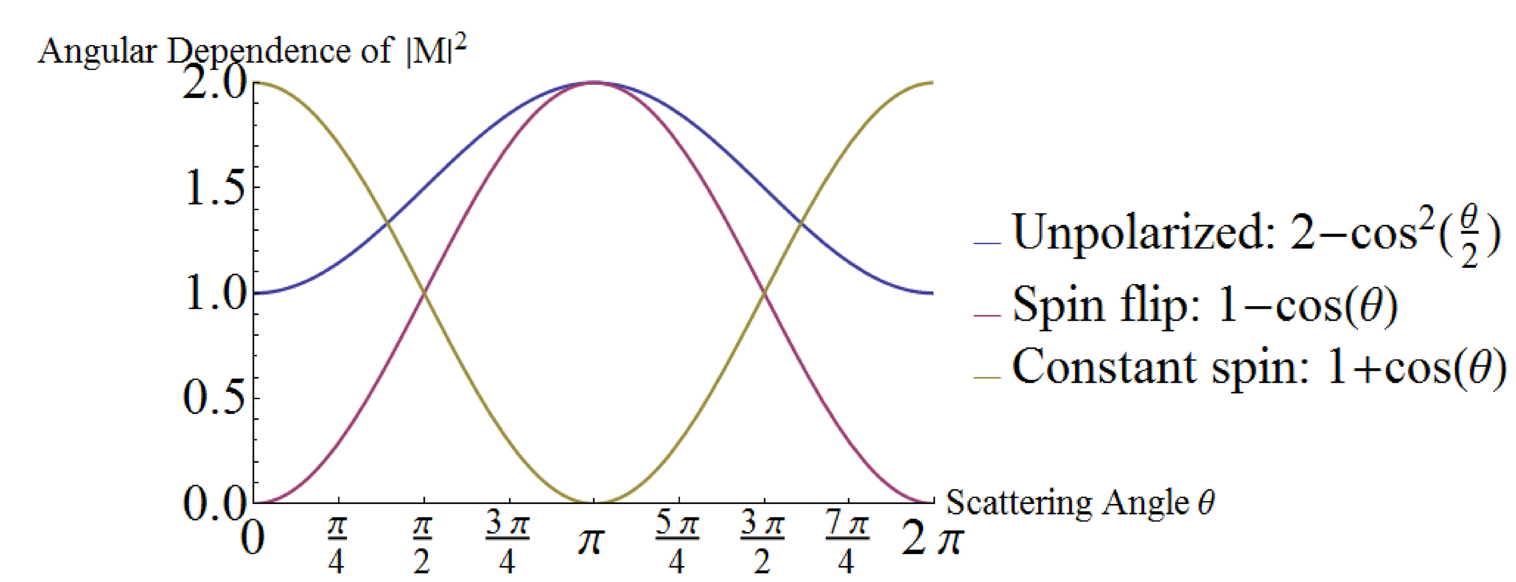
Figure 4

Results

To analyze the angular dependence of the interaction, this amplitude is squared and simplified with initial conditions. For this research, Majorana particles were assumed to be massive relative to the electron (as fits with the leading candidates for dark matter), and all polarization cases were considered, yielding different angular dependences. The differential cross-section $\frac{d\sigma}{d\Omega}$ is proportional to the square of the scattering amplitude $\langle|\mathcal{M}|^2\rangle$ which is contained in the table below.

Polarization of Anapole (initial → final)	Amplitude squared $\langle\|\mathcal{M}\|^2\rangle$
Unpolarized	$64\left(g_e f_A\left(q^2\right)m_\chi c\right)^2 k^2\left(2-\cos^2\frac{\theta}{2}\right)$
Spin up → Spin down	$4\left(g_e f_A\left(q^2\right)m_\chi c\right)^2 k^2(1-\cos\theta)$
Spin down → Spin up	$4\left(g_e f_A\left(q^2\right)m_\chi c\right)^2 k^2(1-\cos\theta)$
Spin up → Spin up	$2\left(g_e f_A\left(q^2\right)m_\chi c\right)^2 k^2(1+\cos\theta)$
Spin down → Spin down	$2\left(g_e f_A\left(q^2\right)m_\chi c\right)^2 k^2(1+\cos\theta)$

Table 1: Results for the amplitude squared for anapole-electron interactions



Conclusion

The most surprising element of these results is the peak amplitude at $\theta = \pi$ for the unpolarized and spin flip cases. This behavior will be further investigated with more analysis of the anapole-electron amplitude, as well as an investigation into anapole-photon scattering. I will continue this research throughout the academic year as a senior thesis.

Acknowledgements

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