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Analogue of the Fizeau effect in an effective optical mediumK. K. Nandi,^{1,2,*} Yuan-Zhong Zhang,^{2,3,†} P. M. Alsing,^{4,‡} J. C. Evans,^{5,§} and A. Bhadra^{6,||}¹*Department of Mathematics, University of North Bengal, Darjeeling WB 734430, India*²*CCAST (World Laboratory), P. O. Box 8730, Beijing 100080, China*³*Institute of Theoretical Physics, Chinese Academy of Sciences, P. O. Box 2735, Beijing 100080, China*⁴*Albuquerque High Performance Computing Center, University of New Mexico, Albuquerque, New Mexico 87131*⁵*Department of Physics, University of Puget Sound, Tacoma, Washington 98416*⁶*High Energy and Cosmic Ray Research Center, University of North Bengal, Darjeeling, WB 734430, India*

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Using a new approach, we propose an analogue of the Fizeau effect for massive and massless particles in an effective optical medium derived from the static, spherically symmetric gravitational field. The medium is naturally perceived as a dispersive medium by matter de Broglie waves. Several Fresnel drag coefficients are worked out, with appropriate interpretations of the wavelengths used. In two cases, it turns out that the coefficients become independent of the wavelength even if the equivalent medium itself is dispersive. A few conceptual issues are also addressed in the process of derivation. It is shown that some of our results complement recent work dealing with real fluid or optical black holes.

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I. INTRODUCTION AND REAPPRAISALS

The historical Fizeau effect for light in moving media has been reconsidered by several authors [1–5] in recent times. We shall consider it here in the context of static, isotropic gravity. To our knowledge, such an investigation has not been undertaken before. We deemed it worthwhile to examine how an old effect would look in a new theoretical model and what conceptual issues are involved. However, we must make clear at the outset that the *only* quantity to be borrowed from general relativity is the effective refractive index. The rest of the analysis is special relativistic (see Sec. III).

In the literature, generally, the Fizeau effect is considered in connection with its close relative, the analogue of the Aharonov-Bohm (AB) effect in a real material medium. Several interesting results have followed from these analyses. For instance, Leonhardt and Pivnicki [6] showed that a non-uniformly moving medium appears to light as an effective gravitational field for which the curvature scalar is nonzero. They also showed how light propagation at large distances around a vortex core shows the Aharonov-Bohm effect and at shorter distances resembles propagation around what are termed optical black holes. Berry *et al.* [7] demonstrated the AB effect with water waves and Roux *et al.* [8] observed it for acoustical waves in classical media. The curved space analogy has been predicted for fluids and superfluids as well [9]. The spirit of the present work, in some sense, is in a direction that is the reverse of the idea of the above curved space analogy. That is, our interest is to calculate the Fizeau type effect for *both* massless and massive particles in a static, spherically symmetric gravity field but portraying it as an

effective optical medium. In the process, we shall also see the extent to which the curved space analogy compares with the results derived in a genuine gravitational field. For simplicity, we shall assume only uniform motion of our effective medium resulting from the relative motion between the gravitating source and the observer.

An outline of the Fizeau effect is this. Consider a tube through which a fluid with a refractive index n is flowing with velocity V . Then, let light pass through the tube parallel to its axis. In the comoving frame of the water, the speed of light is v' ($=c_0/n$), but in the frame in which the water appears to be flowing, the speed of light has been found to be

$$v = v' \pm \left(1 - \frac{v'^2}{c_0^2}\right)V + O(V^2) \approx \frac{c_0}{n} \pm \left(1 - \frac{1}{n^2}\right)V, \quad (1)$$

where c_0 is the speed of light in vacuum. The quantity $(1 - n^{-2})$ is called the Fresnel drag as he was the first to predict it theoretically. Obviously, the resultant speed v does not conform to the Galilean law of addition of velocities $v' \pm V$. The effect, after it was experimentally observed by Fizeau in 1851, was regarded as an empirical fact awaiting a correct theoretical interpretation. It came only after the advent of Einstein's special theory of relativity in 1905. It has since been realized that the Fizeau effect symbolizes only a first order approximation of the exact one-dimensional special relativistic velocity addition law (VAL) derived from Lorentz spacetime transformations. Originally, Fizeau did not consider dispersion but nowadays it is recognized that the effect also contains a term due to the effect of dispersion.

In our investigation, we shall adopt an approach involving quantum mechanics and general and special relativity using the method of what is known as the optical-mechanical analogy. The historical and fundamental role of the analogy in the development of modern theoretical physics need not be emphasized. Apart from the crucial role it played in the development of quantum mechanics, especially in the de Bro-

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glie wave-particle duality, it provides an excellent tool that enables one to visualize the problems of geometrical optics as problems of classical mechanics and vice versa.

In a series of papers [10], it has been shown that the optical-mechanical analogy can be recast into a familiar form that allows one to envisage the mechanical particle equation as a geometrical optical ray equation and the latter as a Newtonian $F=ma$ equation:

$$\frac{d^2\vec{r}}{dA^2} = \vec{\nabla} \left(\frac{1}{2} n^2 c_0^2 \right), \tag{2}$$

$$dA = \frac{dt}{n^2}, \tag{3}$$

where $\vec{r} \equiv (x, y, z)$ or (r, θ, φ) , $\vec{\nabla}$ is the gradient operator, n is the index of refraction, not necessarily constant, and A was originally called the stepping parameter but could also be identified as the optical action and related to several other physical parameters. Many illustrations in ordinary gradient index optics demonstrated the validity of Eqs. (2) and (3) and their usefulness as a heuristic tool.

An interesting turn in the direction of investigation was signaled by the introduction of general relativity [11]. Exact equations for light propagation in the static, spherically symmetric field of Schwarzschild gravity do indeed follow from Eqs. (2) and (3) when an appropriate gravitational index of refraction $n(\vec{r})$ is employed. The analysis also brings forth the distinct but complementary roles played by the optical action A and coordinate time t . To see this, note that the first integral of Eq. (2) is

$$\left| \frac{d\vec{r}}{dA} \right| = n c_0, \tag{4}$$

or equivalently, using Eq. (3),

$$\left| \frac{d\vec{r}}{dt} \right| = \frac{c_0}{n}. \tag{5}$$

However, the force laws have changed thereby. In Eq. (4), the potential is $\frac{1}{2} n^2 c_0^2$ while in Eq. (5) the potential is $-\frac{1}{2} c_0^2 / n^2$. On eliminating A from Eq. (4) or t from Eq. (5), we therefore obtain two path equations for light on a plane, but *only* the former, not the latter, gives the right answer. On the other hand, Eq. (5) gives the correct equation for the Shapiro time delay Δt , while ΔA from Eq. (4) does not. A

deeper understanding of the parameter A is still awaited.¹

Our basic strategy is to regard the gravity field as an effective refractive optical medium imposed on a *fictitious* Minkowski space so that Lorentz transformations can be used to relate two relatively moving observers in that space. (Note that we are not talking of a division of the metric tensor into two parts, but rather of a scalar field placed upon a flat space. For more discussion, see Sec. III.) This is just an intermediate exercise. The final outcome has to be translated back into actually observable quantities in a gravity field. The idea that a gravity field could be formally equivalent to a refractive medium with respect to optical propagation is not new. It goes all the way back to Eddington [12] who was the first to advance the expression of a gravitational refractive index in an approximate form. It was used later, in varying degrees, by several other researchers [13,14] in the investigation of specific problems. But none of the work really focused on how the *exact* general relativistic equations of trajectories, frequency shifts, or Shapiro time delay for massless particles could be obtained in that equivalent medium. The motion of massive particles was not addressed at all. An extension of the work in Ref. [11] that also includes the massive particle motion now exists [15,16]: A suitably modified index of refraction together with the $F=ma$ formulation immediately reproduce all the desired exact equations in the static, isotropic gravity field. The method has been applied very successfully to Friedmann cosmologies as well, which yielded some new interesting insights. All the above systematic developments amply indicate the usefulness of the concept of an effective gravitational index of refraction. By way of a further extension, the index has been calculated also for a more general class of rotating metrics [17]. A new and significant development has come in the shape of the most recent formulation [18] of a single set of unified optical-mechanical equations that allow easy introduction of quantum relations into the index. As a consequence, one then finds that massive de Broglie waves necessarily perceive the gravity field as a *dispersive* optical medium.

In this paper, our basic aim hinges around calculating the consequences arising out of this dispersion in the form of what may be termed the gravitational Fresnel drag, *disper-*

¹An interesting identification of A is that it is simply the affine parameter λ of the null geodesic in the optical metric $ds^2 = -(c^2/n^2)dt^2 + dx^2 + dy^2 + dz^2$. To see this, note that the timelike Killing vector is $K^a = (1, 0, 0, 0)$, while it is a standard result that the inner product between the Killing vector and the tangent vector of a geodesic is conserved, provided the tangent vector is normalized using an affine parameter. That implies $K^a g_{ab} K^b = \text{const}$. That is, $(-c^2/n^2)(dt/d\lambda) = \text{const}$, or $d\lambda$ is proportional to dt/n^2 . This implies that $d\lambda$ is proportional to dA . Of course, this is still a formal mathematical statement, but the fact that the parameter A is related in this way to a null affine parametrization in the sense of general relativity is quite promising. In this context, we should mention that we gave, in Ref. [18], four more relations connecting A with other quantities—proper time, the integration measure in the WKB expansions, Born and Wolf's parameter τ , and phase speed times group displacement. All these issues require a more detailed investigation.

sion included. There are several spin-offs. It will be demonstrated that, in the comoving frame, the expressions for the Lagrangian and the dispersion relation are similar to those obtained by Leonhardt and Piwnicki [6] in the context of real media. These similarities provide a direct extension of these expressions in a realistic gravity field. It will also be evident that the conditions for optical black holes [6,19,20] are naturally met in the equivalent medium, irrespective of whether one considers light or massive de Broglie waves.

The paper is organized as follows. Section II contains a brief survey of the basic equations that will be used throughout the paper. Conceptual justifications for the adopted procedure appear in Sec. III. In Secs. IV–VI, the gravitational Fresnel drags are calculated for different choices of the wavelengths. Section VII contains a brief discussion of operational definitions. In Sec. VIII, we demonstrate how the results dealing with a real fluid medium compare with those in a genuine gravity field considered in this paper. Finally, in Sec. IX, we summarize and add some remarks.

II. BASIC EQUATIONS

Consider a static, spherically symmetric, but not necessarily vacuum, solution of general relativity written in isotropic coordinates

$$ds^2 = \Omega^2(\vec{r}')c_0^2 dt'^2 - \Phi^{-2}(\vec{r}')|d\vec{r}'|^2, \quad (6)$$

where Ω and Φ are the solutions of Einstein's field equations. Many metrics of physical interest can be put into this isotropic form including the experimentally verified Schwarzschild metric. The coordinate speed of light $c(\vec{r}')$ is determined by the condition that the geodesic be null ($ds^2 = 0$):

$$c(\vec{r}') = \left| \frac{d\vec{r}'}{dt} \right| = c_0 \Phi(\vec{r}') \Omega(\vec{r}'). \quad (7)$$

We take leave from the metric approach at this point and define the effective index of refraction for light in the gravitational field as

$$n(\vec{r}') = \Phi^{-1} \Omega^{-1}. \quad (8)$$

We shall omit further details here that can be found in Ref. [18], and only state the results to be used in this paper. The first step in the direction of introducing quantum mechanics in a semiclassical way is to have a single refractive index N and a single set of equations that should be valid for both massless and massive particles. The result is

$$\frac{d^2 \vec{r}'}{dA^2} = \vec{\nabla} \left(\frac{1}{2} N^2 c_0^2 \right) \quad (\text{light and particles}), \quad (9)$$

$$\left| \frac{d\vec{r}'}{dA} \right| = N c_0 \quad (\text{light and particles}), \quad (10)$$

where, once again, it is the same A , satisfying $dA = dt/n^2$, that appears even for massive particle trajectories. It looks as

if the action has a foot in the wave regime and a foot in the particle regime. The second step involves the introduction of the Planck relation $H' = \hbar \omega'$ and the de Broglie relation $p' = \hbar k' = h/\lambda'$, where $h = 2\pi\hbar$, in the expression for N . As usual, H' and p' are the total energy and momentum, respectively, and ω' ($\equiv 2\pi\nu'$) and λ' are the coordinate frequency and the wavelength of the de Broglie waves. The physically measurable corresponding proper quantities are $\tilde{\omega}' = \omega'/\Omega$ and $\tilde{\lambda}' = \lambda'/\Phi$, respectively. The third step finally gives the desired index of refraction N of the dispersive medium due to massive de Broglie waves:

$$N(\vec{r}', \omega') = n(\vec{r}') \sqrt{1 - \frac{m^2 c_0^4 \Omega^2(\vec{r}')}{\hbar^2 \omega'^2}}, \quad (11)$$

where m is the rest mass of the test particle. One may also rewrite N as

$$N = \frac{c_0 p'}{H'} = \frac{n^2 v'}{c_0}, \quad (12)$$

where v' is the (unobservable) coordinate speed of the classical particle in the medium. It also follows that

$$\omega' = 2\pi\nu', \quad \lambda' = \frac{c_0}{N\nu'}. \quad (13)$$

Using Eq. (13), N can be rewritten in a more transparent form:

$$N = \frac{n(\vec{r}')}{\sqrt{1 + (\tilde{\lambda}'/\lambda_c)^2}}, \quad (14)$$

where $\lambda_c = h/mc_0$ is the Compton wavelength of the particle. Clearly, for light, $m = 0, N = n$, and one recovers Eqs. (2) and (3) from Eqs. (9) and (10), respectively. That is, light waves do not perceive the effective medium as dispersive. However, for $m \neq 0$, dispersion seems inevitable, as evidenced from Eq. (11) or (14), if quantum relations are introduced.

We shall require also the following. The mass shell constraint is given by [18]

$$\hbar^2 \omega'^2 = m^2 c_0^4 \Omega^2 + \frac{c_0^2 \hbar^2 k'^2}{n^2}. \quad (15)$$

The phase velocity is

$$v'_p = \frac{H'}{p'} = \frac{\omega'}{k'} = \frac{c_0}{N}, \quad v'_p v'_g = \frac{c_0^2}{n^2}, \quad (16)$$

giving the group velocity

$$v'_g = \frac{d\omega'}{dk'} = \frac{c_0 N}{n^2} = v'. \quad (17)$$

It should be mentioned that the validity of the expression (11) is established also by the WKB analysis of the massive

generally covariant Klein-Gordon equation [18]. Moreover, the mass shell constraint (15) yields the *exact* Stodolsky phase [21] in the case of the spin-1/2 Dirac equation in curved spacetime [22]. This last result is extremely interesting.

With Eqs. (6)–(17) at hand, we are able to calculate the Fresnel drag factors under different scenarios, but, before this, we need to clear up a few relevant concepts. Note that all the expressions in this section refer to the comoving frame, that is, the frame fixed to the gravitating source. Henceforth, in order to have conformity with notation in the literature, all expressions in the comoving frame will be designated by primes and those in the relatively moving lab frame will be unprimed.

III. CONCEPTUAL ISSUES

The following discussion is aimed at providing appropriate interpretations of the quantities that appear in the various formulations of the Fizeau effect. There are two basic ingredients. The first is the VAL. In many works dealing with the effect, the one-dimensional VAL, which is valid for point particles, is also employed, implicitly or explicitly, for *waves* propagating with the phase speed c_0/n . The procedure is to use the one-dimensional Lorentz transformation equations in the form

$$\omega' = \gamma(\omega - kV), \quad k' = \gamma(k - V\omega c_0^{-2}), \quad (18)$$

$$\omega = \gamma(\omega' + k'V), \quad k = \gamma(k' + V\omega' c_0^{-2}),$$

$$\gamma = (1 - V^2/c_0^2)^{-1/2}, \quad (19)$$

and obtain a VAL as

$$v'_p = (v_p - V)(1 - v_p V/c_0^2), \quad v_p = (v'_p + V)(1 + v'_p V/c_0^2), \quad (20)$$

where $v'_p = \omega'/k' = c_0/n$ is the phase speed of light in the (primed) comoving frame of the medium and $v_p = \omega/k$ is the phase speed in the (unprimed) lab frame in which the medium appears to be moving with uniform relative velocity V . The phase speed, however, could well exceed c_0 in many physical configurations where $n < 1$.

On the other hand, an *a priori* prescription that n be greater than unity (making $c_0/n < c_0$) somewhat diminishes the generality of the theory. However, this deficiency may not pose any realistic problem in a nondispersive medium. When dispersion is involved, the most appropriate quantity to use in the VAL is the group speed $d\omega/dk$ (which involves a knowledge of $dn/d\omega$), which simply equals the velocity of the classical point particle, rather than the phase speed. As stated before, the original Fizeau experiment did not consider any dispersion; the index n was taken to be a true constant, so that the group and the phase velocities coincided precisely at c_0/n . In general, they are different for massive de Broglie waves, as our later equations will reveal. In our calculation of the Fizeau effect, the mass shell constraint Eq. (15), or, by another name, the dispersion relation, plays a key role: It provides well defined expressions for the group and phase

velocities. Such types of natural constraints are unavailable in just any arbitrary medium consisting of solids or liquids. In these cases, dispersion is normally introduced by hand.

An important point should be noted here. In describing the Fizeau experiment with an ordinary medium (such as water), one takes the background spacetime to be flat. Such Minkowski networks, composed of rods and clocks, are actually unobservable in a gravity field due to the universality of gravitational interaction, or, putting it more technically, due to the principle of equivalence. There does not exist a unique division of the metric tensor into a background and a field part. We consider here a different kind of separation according to which the gravity field is looked upon as analogous to an optical medium imposed upon a flat background spacetime, the index N summarizing the nonlinearities of the gravity field, as it were. The important point is that the analogy, although intended to be only of *formal* nature, may lead to results that could be testable by experiment (see Sec. IX for a discussion). With this understanding, let us conceive of observers equipped with fictitious Minkowski networks and apply, as an intermediate step, the full machinery of special relativity in what follows.

Thus, we take Eq. (19) in the form

$$\Delta\omega = \gamma(\Delta\omega' + \Delta k'V), \quad \Delta k = \gamma(\Delta k' + V\Delta\omega' c_0^{-2}), \quad (21)$$

which gives the VAL, denoting $v'_g = \Delta\omega'/\Delta k'$, as

$$v_g = \frac{\Delta\omega}{\Delta k} = \frac{v'_g + V}{1 + Vv'_g/c_0^2}. \quad (22)$$

The second ingredient is the special relativistic Doppler shift in one dimension giving the frequency (or wavelength) transformation between two frames in relative motion. Thus, one takes Eq. (19) in the form

$$\omega = \gamma\omega'(1 + k'V/\omega'), \quad (23)$$

and specifies ω'/k' . At this point, let us note that Cook, Fearn, and Milonni [2] considered two possibilities in the context of a Fizeau experiment with real media having refractive indices n .

A. Case 1

Take $\omega'/k' = c_0$ in Eq. (23). This case was considered by Synge [3]. That is, take the usual Doppler shift formula, which, written in terms of the wavelength, is

$$\lambda = \lambda' \sqrt{\frac{1 - V/c_0}{1 + V/c_0}}. \quad (24)$$

The corresponding physical configuration consists of a block of material moving with velocity V in an otherwise empty lab frame. The wavelength λ' of a light pulse measured by an observer stationed at the interface between the block and the empty space will appear to the lab observer as λ according to Eq. (24). Inside the block, however, λ' is assumed to

be a constant. The resulting Fresnel drag has been experimentally confirmed to a very good accuracy by Sanders and Ezekiel [4].

Unfortunately, it is difficult to conceive of a parallel configuration in our problem. The entire optical medium cannot be simply put inside a box with a certain boundary, nor need the wavelength λ' be constant throughout the medium. Instead, it is easier to consider two relatively moving observers associated with the background empty frame who may use Eq. (24). We have to calculate how one observer translates the observations of another at a certain point when they happen to pass each other. This is done in Sec. IV.

B. Case 2

Take $\omega'/k' = c_0/n$ in Eq. (19) for k . Cook *et al.* [2] provide the corresponding physical configuration in this case. According to Lerche [1], the lab observer can exercise two options. He/she either uses (i) a wavelength λ given by the Doppler formula (23) but with $\omega'/k' = c_0/n$ or uses (ii) a vacuum wavelength $\lambda_0 = 2\pi c_0/\omega$. The forms for the drag coefficients will be different in the two cases. The parallel options in our case are the same, except that we have to use N instead of n , so that the Doppler formula reads

$$\lambda = \frac{\lambda'}{\gamma} \left(1 + \frac{V}{Nc_0} \right)^{-1}. \quad (25)$$

We shall work out both the options in Sec. V. This particular formula appears to be more consistent with our formulation *per se* as we will be using our own definition of ω'/k' given in Eq. (27). We can also add a *third* possibility worked out in Sec. VI. This is a special feature of the gravitational case we are considering.

C. Case 3

Consider a stationary observer \tilde{A} at a point in the gravity field measuring the proper (or physical) wavelength $\tilde{\lambda}'$. He/she also measures the proper velocity of light in his/her neighborhood to be just c_0 . A freely falling observer \tilde{B} at that point, having an instantaneous velocity \tilde{V} relative to \tilde{A} , would measure $\tilde{\lambda}$ according to the options, which, to first order, are

$$\tilde{\lambda}' \approx \tilde{\lambda}(1 + \tilde{V}/c_0), \quad \tilde{\lambda}' \approx \tilde{\lambda}(1 + \tilde{V}\tilde{N}/c_0), \quad \tilde{\lambda}_0 = 2\pi c_0/\tilde{\omega}. \quad (26)$$

Note that there is a difference between the present stationary observer and the stationary observer associated with the background flat space of case 1: The group velocities of the matter de Broglie waves measured by them are not the same (see below). We now proceed to calculate the Fresnel drags successively in all three cases using the same VAL Eq. (22), but different Doppler formula, Eqs. (24)–(26).

IV. FRESNEL DRAG: CASE 1

Suppose that an observer A , equipped with a Minkowski network, is at rest at $r=0$ in a spherically symmetric me-

dium. He/she measures the coordinate phase and group velocities of a massive de Broglie wave packet at $r=r'_0$ as, using Eq. (15),

$$\frac{\omega'}{k'} = v'_p = \frac{c_0}{N(r'_0, \lambda')}, \quad (27)$$

$$\frac{\Delta\omega'}{\Delta k'} = v'_g = \frac{c_0}{\bar{N}(r'_0, \lambda')} < c_0, \quad \bar{N} = \frac{n^2}{N} > 1. \quad (28)$$

For a light pulse, $v'_p = v'_g = c_0/n$ and these are independent of the wavelength λ' or wave number k' . The same holds for v_g as well. This implies that the trajectories of light rays do not depend on the wave properties of light. However, in general, $v'_p \neq v'_g$, as is evident from Eqs. (27) and (28).

Consider another observer B moving in the same radial direction with uniform velocity V with respect to A . Then, in the frame of B , identified as the lab observer, the entire medium moves uniformly, that is, A becomes the comoving observer. How will B translate the observations of A , when their origins coincide at $r=0$? To find out, note that the coordinate length r'_0 will appear to B as

$$r_0 = r'_0 \sqrt{1 - \frac{V^2}{c_0^2}}. \quad (29)$$

Also, the velocity v'_g observed by A will appear to B as v_g given by the special relativistic VAL Eq. (22). We may explicitly express v'_g in terms of (r'_0, λ') as

$$v'_g = \frac{c_0}{\bar{N}(r'_0, \lambda')} = \frac{c_0}{n(r'_0) \times \sqrt{1 + (\lambda'/\lambda_c)^2 \Phi^{-2}(r'_0)}}. \quad (30)$$

When this expression for v'_g is plugged into the right hand side of Eq. (22), one finds the answer to the question above: $v_g(r'_0, \lambda')$ is the exact radial group velocity of the de Broglie waves to be observed by B . But B uses the Doppler shifted wavelength λ instead of λ' . Then, to first order in (V/c_0) , we get from Eq. (24)

$$\lambda' \approx \lambda(1 + V/c_0) = \lambda + \Delta\lambda, \quad r'_0 \approx r_0. \quad (31)$$

Considering the right hand side of Eq. (30) and writing the denominator as $\bar{N}(r'_0 = r_0, \lambda') \equiv \bar{N}(\lambda + \Delta\lambda)$, we get from the Taylor expansion

$$\bar{N}(\lambda + \Delta\lambda) \approx \bar{N}(\lambda) + \Delta\lambda \frac{\partial \bar{N}}{\partial \lambda} = \bar{N}(\lambda) \left(1 + \frac{\lambda V}{c_0 \bar{N}} \frac{\partial \bar{N}}{\partial \lambda} \right).$$

From Eqs. (22) and (28), we get, using the above, a redefined index \bar{N}' such that

$$v_g(\lambda') = \frac{c_0}{\bar{N}'(\lambda')} = \frac{c_0}{\bar{N}(\lambda')} + \left(1 - \frac{1}{\bar{N}^2(\lambda')} \right) V. \quad (32)$$

In other words, in the approximation considered, $\bar{N}^2(\lambda')$ $\approx \bar{N}^2(\lambda)$ and we have

$$v_g(\lambda) = \frac{c_0}{\bar{N}(\lambda + \Delta\lambda)} + \left(1 - \frac{1}{\bar{N}^2(\lambda)}\right)V = \frac{c_0}{\bar{N}(\lambda)} + F_1 V, \quad (33)$$

where

$$F_1 \equiv \left(1 - \frac{1}{\bar{N}^2(\lambda)}\right) - \frac{\lambda}{\bar{N}^2(\lambda)} \times \frac{\partial \bar{N}}{\partial \lambda} \quad (34)$$

is the Fresnel drag we have been looking for. It can be easily verified that the same F_1 follows also from the ordinary expansion of $v_g(r'_0, \lambda')$ in Eq. (22) in conjunction with Eqs. (21) and (30) under the small velocity approximation, Eq. (31), but the steps as given above are the simplest. For light waves, $\bar{N} \rightarrow n$, and one has

$$F_1 \equiv \left(1 - \frac{1}{n^2(\lambda)}\right) - \frac{\lambda}{n^2(\lambda)} \times \frac{\partial n}{\partial \lambda}. \quad (35)$$

Interestingly, although the dependence of n on λ is not known, the dispersion nonetheless follows here as an inheritance from Eq. (34). This is the formula proposed by Synge [3] and also experimentally tested [4] with n as the refractive index of the block.

Using Eqs. (14) and (28), we can have the explicit expression from Eq. (34) as

$$F_1 = 1 - \frac{1}{n^2(r_0)[1 + (\tilde{\lambda}/\lambda_c)^2]} - \frac{(\tilde{\lambda}/\lambda_c)^2}{n(r_0)[1 + (\tilde{\lambda}/\lambda_c)^2]^{3/2}}. \quad (36)$$

Note that, in the asymptotic region $r \rightarrow \infty$, or in the absence of gravity, one has $n(r) \rightarrow 1$, $\tilde{\lambda} \rightarrow \lambda'$, so that, from Eq. (28), the group and phase velocities of de Broglie waves, as measured by A , are, respectively,

$$v'_g = v' = \frac{c_0}{[1 + (\lambda'/\lambda_c)^2]^{1/2}} < c_0, \quad (37)$$

$$v'_p = c_0 \left[1 + \left(\frac{\lambda'}{\lambda_c}\right)^2\right]^{1/2} > c_0,$$

and thus one finds that matter de Broglie waves perceive even the flat space as a dispersive medium with an index of refraction

$$\bar{N}_{flat} = \left[1 + \left(\frac{\lambda'}{\lambda_c}\right)^2\right]^{1/2}. \quad (38)$$

One recognizes that it is this v'_g in Eq. (37), together with $\lambda'/\lambda_c = mc_0/p'$, that provides the energy transformation law:

$$H = \frac{mc_0^2}{\sqrt{1 - v_g'^2/c_0^2}}. \quad (39)$$

Then one recovers the special relativistic mass shell condition. It follows that, in this case, the drag measured by B in terms of his/her wavelength λ is

$$F_1^{flat} = \frac{(\lambda/\lambda_c)^2}{[1 + (\lambda/\lambda_c)^2]} \times \left[1 - \frac{1}{[1 + (\lambda/\lambda_c)^2]^{1/2}}\right]. \quad (40)$$

As one can see, Eqs. (37)–(40) are restatements of the well known special relativistic expressions, only *interpreted* in a different way.

V. FRESNEL DRAG: CASE 2

According to the first option (i) in Sec. III B the Doppler shift is given by Eq. (25). Thus, we have, to first order in (V/c_0),

$$\lambda' \approx \lambda(1 + V/Nc_0) = \lambda + \Delta\lambda, \quad \Delta\lambda = \frac{\lambda V}{Nc_0}. \quad (41)$$

Then, writing again $\bar{N}(r'_0, \lambda') \equiv \bar{N}(\lambda + \Delta\lambda)$, we get from the Taylor expansion

$$\bar{N}(\lambda + \Delta\lambda) \approx \bar{N}(\lambda) + \Delta\lambda \frac{\partial \bar{N}}{\partial \lambda} = \bar{N}(\lambda) \left(1 + \frac{\lambda V}{c_0 N \bar{N}} \frac{\partial \bar{N}}{\partial \lambda}\right). \quad (42)$$

The resultant group velocity as observed by B , who uses λ of Eq. (41), is

$$v_g = \frac{c_0}{\bar{N}'(\lambda')} = \frac{c_0}{\bar{N}(\lambda')} + \left(1 - \frac{1}{\bar{N}^2(\lambda')}\right)V$$

$$= \frac{c_0}{\bar{N}(\lambda + \Delta\lambda)} + \left(1 - \frac{1}{\bar{N}^2(\lambda)}\right)V$$

$$= \frac{c_0}{\bar{N}(\lambda)} + F_2 V, \quad (43)$$

where

$$F_2 \equiv \left(1 - \frac{1}{\bar{N}^2(\lambda)}\right) - \frac{\lambda}{N \bar{N}^2(\lambda)} \times \frac{\partial \bar{N}}{\partial \lambda} \quad (44)$$

is the drag factor. For light waves, $N = n$, $\bar{N} = n$ so that

$$F_2 \equiv \left(1 - \frac{1}{n^2(\lambda)}\right) - \frac{\lambda}{n^3(\lambda)} \times \frac{\partial n}{\partial \lambda}. \quad (45)$$

This formula was first given by McCrea [23]. Writing explicitly, we find, from Eq. (44),

$$F_2 = 1 - \frac{1}{n^2(r_0)}. \quad (46)$$

This coefficient comes out to be independent of λ . According to the second option (ii), B uses a vacuum wavelength. In this case, the calculations would proceed slightly differently. Consider Eq. (18) for ω' instead of Eq. (41). Then we have, to first order,

$$\omega' \approx \omega(1 - VN(\omega)/c_0) = \omega + \Delta\omega, \quad \Delta\omega = -\frac{\omega VN(\omega)}{c_0}. \quad (47)$$

Then, proceeding as before,

$$v_g(\omega) = \frac{c_0}{\bar{N}(\omega)} + \left[\left(1 - \frac{1}{\bar{N}^2(\omega)} \right) + \frac{\omega}{\bar{N}(\omega)} \times \frac{\partial \bar{N}}{\partial \omega} \right] V. \quad (48)$$

Now B uses the vacuum wavelength as $\lambda_0 = 2\pi c_0/\omega$, so that Eq. (48) gives

$$v_g(\lambda_0) = \frac{c_0}{\bar{N}(\lambda_0)} + F_3 V, \quad (49)$$

where

$$F_3 = \left[\left(1 - \frac{1}{\bar{N}^2(\lambda_0)} \right) - \frac{\lambda_0 N(\lambda_0)}{\bar{N}^2(\lambda_0)} \times \frac{\partial \bar{N}}{\partial \lambda_0} \right]. \quad (50)$$

For light waves, we get

$$F_3 \equiv \left(1 - \frac{1}{n^2(\lambda_0)} \right) - \frac{\lambda_0}{n(\lambda_0)} \times \frac{\partial n}{\partial \lambda_0}. \quad (51)$$

This is the expression given by Lerche [1] and Cook *et al.* [2] for a Fizeau experiment with water with index n . Writing explicitly, we find from Eq. (50)

$$F_3 = 1 - \frac{1}{n^2(r_0)[1 + (\tilde{\lambda}_0/\lambda_c)^2]} - \frac{(\tilde{\lambda}_0/\lambda_c)^2}{[1 + (\tilde{\lambda}_0/\lambda_c)^2]^2} \quad (52)$$

where $\tilde{\lambda}_0 = \lambda_0 \Phi^{-1}$. Thus, so far, corresponding to N and \bar{N} , we have three Fresnel coefficients F_1 , F_2 , and F_3 depending on the VAL and the various Doppler shifted wavelengths used by B , as considered in the literature.

VI. FRESNEL DRAG: CASE 3

Consider an observer \tilde{A} at rest with respect to the gravitating source at a coordinate radial distance $r = r'_0$. He/she will measure proper quantities. The mass shell condition is given by

$$\hbar^2 \tilde{\omega}'^2 = m^2 c_0^4 + c_0^2 \hbar^2 \tilde{k}'^2, \quad \tilde{k}' = \Phi k', \quad (53)$$

so that \tilde{A} measures, in his/her neighborhood, the proper phase and group velocities of the de Broglie waves which are connected by

$$\tilde{v}'_p \tilde{v}'_g = \frac{\tilde{\omega}'}{\tilde{k}'} \frac{d\tilde{\omega}'}{d\tilde{k}'} = c_0^2, \quad (54)$$

where, using Eq. (27),

$$\tilde{v}'_p = \frac{\tilde{\omega}'}{\tilde{k}'} = n \left(\frac{\omega'}{k'} \right) = c_0 \bar{N}(r'_0, \tilde{\lambda}') > c_0,$$

$$\tilde{v}'_g = \frac{d\tilde{\omega}'}{d\tilde{k}'} = \frac{c_0}{\bar{N}(r'_0, \tilde{\lambda}')} < c_0,$$

$$\bar{N} \equiv \frac{n}{N} > 1. \quad (55)$$

Note that these phase and group velocities are not the same as those measured by A , viz., Eqs. (27) and (28), which highlights the difference between the two observers. The observer \tilde{A} measures the velocity of light as $\tilde{v}'_p = \tilde{v}'_g = c_0$ since $N = n$. Consider another observer \tilde{B} falling freely in the same radial direction attaining an instantaneous speed \tilde{V} at $r = r'_0$. Since the frame in which \tilde{B} is at rest is locally inertial in virtue of the principle of equivalence, the speed of light measured by \tilde{B} will also be c_0 and hence \tilde{A} and \tilde{B} can be connected by a Lorentz transformation. Then \tilde{v}'_g will appear to \tilde{B} at $r = r_0$ as \tilde{v}_g given by the VAL

$$\tilde{v}_g = \frac{\tilde{v}'_g + \tilde{V}}{1 + \tilde{V} \tilde{v}'_g / c_0^2}. \quad (56)$$

Employing arguments similar to those in cases 1 and 2 we can immediately write down the corresponding drag coefficients

(a) \tilde{A} measures $\tilde{\lambda}'$ and \tilde{B} uses $\tilde{\lambda}$ connected by $\tilde{\lambda}' \approx \tilde{\lambda}(1 + \tilde{V}/c_0)$:

$$\begin{aligned} \tilde{F}_1 &\equiv \left(1 - \frac{1}{\bar{N}^2(\tilde{\lambda})} \right) - \frac{\tilde{\lambda}}{\bar{N}^2(\tilde{\lambda})} \times \frac{\partial \bar{N}}{\partial \tilde{\lambda}} \\ &= \frac{(\tilde{\lambda}/\lambda_c)^2}{[1 + (\tilde{\lambda}/\lambda_c)^2]} \times \left[1 - \frac{1}{[1 + (\tilde{\lambda}/\lambda_c)^2]^{1/2}} \right]. \end{aligned} \quad (57)$$

(b) \tilde{A} measures $\tilde{\lambda}'$ and \tilde{B} uses $\tilde{\lambda}$ connected by $\tilde{\lambda}' \approx \tilde{\lambda}(1 + \tilde{V}\bar{N}/c_0)$:

$$\tilde{F}_2 \equiv \left(1 - \frac{1}{\bar{N}^2(\tilde{\lambda})} \right) - \frac{\tilde{\lambda}}{\bar{N}(\tilde{\lambda})} \times \frac{\partial \bar{N}}{\partial \tilde{\lambda}} = 0. \quad (58)$$

(c) \tilde{A} measures $\tilde{\omega}$ and \tilde{B} uses $\tilde{\lambda}_0$ connected by $\tilde{\lambda}_0 = 2\pi c_0/\tilde{\omega}$:

$$\begin{aligned} \tilde{F}_3 &\equiv \left(1 - \frac{1}{\tilde{N}^2(\tilde{\lambda}_0)}\right) - \frac{\tilde{\lambda}_0}{\tilde{N}^3(\tilde{\lambda}_0)} \times \frac{\partial \tilde{N}}{\partial \tilde{\lambda}_0} \\ &= 1 - \frac{1}{[1 + (\tilde{\lambda}_0/\lambda_c)^2]} \times \left[1 + \frac{(\tilde{\lambda}_0/\lambda_c)^2}{[1 + (\tilde{\lambda}_0/\lambda_c)^2]}\right], \end{aligned} \quad (59)$$

where $\tilde{\lambda}_0 = \lambda_0 \Phi^{-1}$. We also see that the radial proper velocity of the classical point particle as measured by \tilde{A} at $r = r'_0$ is given by

$$\tilde{v}'_{prop} = \frac{dl'}{d\tau'} = n \frac{dr'}{dt'} = n v'_{coord}. \quad (60)$$

Using the definitions $dl' = \Phi^{-1} dr'$, $d\tau' = \Omega dt'$, $v'_{coord} = N c_0 / n^2$, we find that $\tilde{v}'_{prop} = \tilde{v}'_g$. For light, of course, $v'_{coord} = c_0 / n$ and $\tilde{v}'_{prop} = \tilde{v}'_g = c_0$. The last result is also consistent with the fact that $ds^2 = c_0^2 d\tau'^2 - dl'^2 = 0$ gives $dl'/d\tau' = c_0$. For light waves, we find $\tilde{N} = 1$, so that $\tilde{F}_1 = \tilde{F}_2 = \tilde{F}_3 = 0$. These indicate only the special relativistic invariance of light speed, no matter what wavelength \tilde{B} uses. For de Broglie waves, the difference among the drag coefficients is evident from Eqs. (57)–(59).

VII. OPERATIONAL DEFINITIONS

In order to operationally realize the value of F_1 in a gravitational field, consider a simple thought experiment. Let there be a source in free space that produces de Broglie waves with wavelength λ' . Then λ is known via Eq. (31); this is the wavelength measured by B . Let A take this source to any point inside the refractive medium. Then, A will measure the same λ' as $\tilde{\lambda}' = \lambda' \Phi^{-1}$ and B will find $\tilde{\lambda} = \lambda \Phi^{-1}$. The only other quantity is the coordinate distance r_0 appearing in the refractive indices $n(r_0)$ and $\Phi(r_0)$. The expression for the index is supplied by the metric functions. For instance, in the Reissner-Nordstrom field, with $G = c_0 = 1$, we have

$$\Omega^2(r) = \left[1 - \frac{(M^2 - Q^2)}{4r^2}\right]^2 \left[1 + \frac{M}{r} + \frac{(M^2 - Q^2)}{4r^2}\right]^{-2}, \quad (61)$$

$$\Phi^{-2}(r) = \left[1 + \frac{M}{r} + \frac{(M^2 - Q^2)}{4r^2}\right]^2, \quad (62)$$

where M and Q are the mass and the electric charge. For the Schwarzschild field, we have $Q = 0$, so that

$$n(r) = \frac{(1 + M/2r)^3}{(1 - M/2r)}. \quad (63)$$

If the relative velocity V between A and B is small, $V \ll c_0$, we can take $r_0 \approx r'_0$ from Eq. (29). If we consider that both the observers are in a weak gravity field, we can take

$r_0 \approx r'_0 \approx l$, where l is the physically measurable distance from the center of the gravitating source to the field point. Then

$$n(r_0) \approx n(l) \approx 1 + \frac{2M}{l}. \quad (64)$$

With these inputs, Eq. (36) provides the theoretically predicted value of F_1 after the known value of the Compton wavelength is plugged in.

Interesting results are obtained in the case of F_2 and \tilde{F}_2 . One finds that F_2 does not involve the wavelength at all. This means that a Fizeau type experiment either with light or with de Broglie waves will yield the same drag factor, if Eq. (23) is followed in conjunction with Eq. (27). In this case, it appears that the wavelength dependence introduced by the group velocity is *undone* by the Doppler shift. A similar thing occurs also in the case of \tilde{F}_2 which is identically zero.

VIII. COMPARISON WITH REAL MEDIUM

Starting from the wave equation in a nonuniformly moving fluid with refractive index n , Leonhardt and Piwnicki [6] derived the Lagrangian and the Hamiltonian for a light ray as observed by a lab observer. From the action principle, they arrived at a completely geometrical picture of ray optics in a moving medium. Light rays are geodesic lines with respect to Gordon's metric, which in the comoving frame reads

$$ds^2 = \frac{c_0^2}{n^2} dt'^2 - |d\vec{r}'|^2. \quad (65)$$

The Lagrangian, Eq. (49) of Ref. [6], that they derived for a light particle in the lab frame is

$$L = -m c_0 \sqrt{c_0^2 - v^2 + \left(\frac{1}{n^2} - 1\right) \gamma^2 \left(c_0 - \frac{\vec{u} \cdot \vec{v}}{c_0}\right)^2}, \quad (66)$$

where u is the fluid velocity in the lab frame, $v[\equiv (v' + u)/(1 + v'u/c_0^2)]$ is the velocity of the light particle conceived of as having a fictitious mass m , and $\gamma^2 = (1 - u^2/c_0^2)^{-1}$. In the comoving frame of the fluid element, $\vec{u} = 0$ so that

$$L = -m c_0^2 \times \frac{1}{n} \times \sqrt{1 - \frac{v'^2 n^2}{c_0^2}}. \quad (67)$$

Consider the Lagrangian for a massive particle in the comoving frame, derived in our Ref. [18], viz.,

$$L = -m c_0^2 \Omega \left[1 - \frac{v'^2 n^2}{c_0^2}\right]^{1/2}, \quad (68)$$

where v' is the classical particle coordinate speed. Now note that the metric (65) with n as the real medium index can be obtained formally from Eq. (6) above simply by putting $\Phi = 1$ and $\Omega = 1/n$. Clearly, the n in Eq. (68) has a *different*

origin: it derives from general relativity. Using this value of Ω in Eq. (68), one finds that it is exactly the same as Eq. (67).

The dispersion relation for light in the comoving frame ($\vec{u}=\vec{0}$) following from Eq. (33) of Ref. [6] is

$$\omega'^2 - c_0^2 k'^2 + (n^2 - 1)\omega'^2 = 0. \quad (69)$$

This is precisely the same as that following from Eq. (15) with $m=0$ for light.

Interestingly, taking a cue from Eq. (66), we may proceed to write down the Lagrangian of the classical particle in the lab frame as follows. Our metric Eq. (6) in the comoving frame can be written down as

$$ds^2 = \Phi^{-2} \times \left[c_0^2 dt'^2 - d\vec{r}'^2 + \left(\frac{1}{n^2} - 1 \right) c_0^2 dt'^2 \right]. \quad (70)$$

To go to the lab frame, we effect a Lorentz transformation. Note that there is a Lorentz invariant term in the parentheses and hence only the last term needs to be transformed. Thus, in the lab frame the metric is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \Phi^{-2} \times \left[\eta_{\mu\nu} + \left(\frac{1}{n^2} - 1 \right) V_\mu V_\nu \right] dx^\mu dx^\nu, \quad (71)$$

where $\eta_{\mu\nu} = [c_0^2, -1, -1, -1]$, $V_\mu = \gamma(1, -\vec{V}/c_0)$, $\gamma = (1 - V^2/c_0^2)^{-1/2}$, and \vec{V} is the velocity of our medium in the lab frame. In the comoving frame, $V_\mu = (1, 0, 0, 0)$. The action is given by

$$S = -mc_0 \int \sqrt{g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt}} dt = \int L dt.$$

Defining $v^\mu = dx^\mu/dt = (1, \vec{v})$, we can find the Lagrangian for a particle in the lab frame,

$$L = -mc_0 \Phi^{-1} \times \sqrt{c_0^2 - v^2 + \left(\frac{1}{n^2} - 1 \right) \gamma^2 \left(c_0 - \frac{\vec{V} \cdot \vec{v}}{c_0} \right)^2}. \quad (72)$$

The dispersion relation (or the Hamiltonian) in the lab frame can also be obtained by a Lorentz transformation on the mass shell equation (15) in the comoving frame, rewritten as

$$\omega'^2 - c_0^2 k'^2 + (n^2 - 1)\omega'^2 = \frac{m^2 c_0^4 n^2 \Omega^2}{\hbar^2}. \quad (73)$$

Note that the right hand side is a Lorentz scalar and the left hand side has a Lorentz invariant part $\omega'^2 - c_0^2 k'^2$. The remaining part can be transformed to give

$$\omega^2 - c_0^2 k^2 + (n^2 - 1)\gamma^2 (\omega - \vec{k} \cdot \vec{V})^2 = \frac{m^2 c_0^4 n^2 \Omega^2}{\hbar^2}, \quad (74)$$

where $k_\mu = (\omega/c_0, -\vec{k})$ is the wave four-vector. There are several other ways in which Eq. (74) could be obtained, either by the usual Legendre transformations from Eq. (72) or by the Hamilton-Jacobi equation $g^{\mu\nu}(\partial S/\partial x^\mu)(\partial S/\partial x^\nu) = m^2 c_0^4$ with $g^{\mu\nu} = \Phi^2 \times [\eta^{\mu\nu} + (n^2 - 1)V^\mu V^\nu]$. We do not do it here.

A further interesting result holds as a corollary to Sec. IV: For light waves in flat space, $v'_p = v'_g = c_0 = v_g$, as expected. It should be noted that the Minkowski observers A and B can also be located in the asymptotic region and the entire analysis would remain the same. From the asymptotic vantage point, these observers can see that, near the horizon, $n \rightarrow \infty$, and thus $v'_p, v'_g \rightarrow 0$, for *both* light and matter de Broglie waves. It is exactly here that we find that the conditions for optical black holes required by Leonhardt and Piwnicki [19] and Hau *et al.* [20] are provided *most naturally*, that is, extremely low group velocity or high refractive index. In this respect, optical and gravitational black holes indeed look similar. Also, $v_g = V$, implying that, while A sees everything standing still at the horizon, B sees them moving away at the speed V because of B 's own relative motion. This is what we should really expect.

IX. SUMMARY AND CONCLUDING REMARKS

The present investigation is inspired by recent discoveries and analyses of light propagation in Bose-Einstein condensates [19,20]. The extremely low velocity of light in such condensates lead to the possibility of creating optical analogues of astrophysical black holes in the laboratory. In order to theoretically model this possibility, Leonhardt and Piwnicki [6] proceeded from the moving optical medium to an effective gravity field with a scalar curvature $R \neq 0$ in which light propagation is shown to mimic that around a vortex core or optical black hole. It has been shown recently that the propagation of photons in a nonlinear dielectric medium can also be described as a motion in an effective spacetime geometry and several interesting results have followed thereby [24]. Our approach here has been in the exact reverse direction: We proceed from the gravity field and arrive at an effective optical refractive medium and examine the theoretical consequences. The motion of this medium is caused by the relative motion between the observer B and the gravitating source.

We must mention that works based on the above mentioned analogies provide some curious theoretical insights both in real media and in the gravitational field, as a result of wisdom borrowed from one field and implanted into the other. This has been the basic philosophy of the present paper. Many more interesting results are known apart from the possibility of optical black holes stated above. For instance, an analysis in acoustic theory leads to the remarkable result that the Hawking radiation in black hole physics is not of dynamical but of *kinematical* origin (Visser, Ref. [9]). Conversely, a gravitational refractive index approach, similar in spirit to ours, has yielded the possibility of *Cerenkov radiation* in the outskirts of a wormhole throat [25–28]. In the present paper, we envisaged a nontrivial *dispersive Fresnel drag* coefficient in a gravity field. We must emphasize that

these results are only of pedagogic interest at present. A further confirmation or otherwise of these results would establish the extent to which these analogies can actually be relied upon.

We saw above how dispersion effects, for both massless and massive particles, appear naturally as a consequence of the systematic development of an effective medium approach to the gravitational field. Various expressions for the drag coefficients result due to the use of VAL and different wavelengths used by the observer B . (See Refs. [1–3] for more detailed arguments on the question of the use of the appropriate wavelength.) It is demonstrated that F_2 is *independent* of λ even in a dispersive medium for massive particles and that \tilde{F}_2 is identically zero. These results may have interesting implications for both optical and general relativity black holes.

It does not seem easy to simulate real experiments, with our type of unbounded medium, that parallel those dealing with ordinary media like solids, fluids, or superfluids. For this reason, we limited ourselves only to theoretical calculations of the drag coefficients, and the expressions may be useful in the study of the passage of light and cosmic particles in astrophysical media, since what we actually see from the moving Earth is not what was originally sent from the source. This work is underway.

We saw that the present analysis naturally complements the curved space analogy of a moving medium. Some of the key expressions in the comoving frame are indeed the same.

Moreover, we can find a direct extension of the expressions to a genuine gravity field (Sec. VIII). The resulting Lagrangian and Hamiltonian describe the trajectories of a particle as viewed from the lab frame, say, a rocket. It also appears that the nomenclature “optical black holes” is quite apt as the conditions required for their creation are most naturally met near the gravitational horizon. This gives an indication that the behavior of the real optical medium should mimic that of our equivalent refractive medium around a coordinate singularity. A favorable situation is attained if light perceives the highly refractive real optical medium as dispersionless which, in our effective medium, is actually the case. Leonhardt and Piwnicki [6] also make a similar statement in the context of their vortex analysis. It is interesting to note that an index of the form $n=C/r$, where C is a constant, when put in Eq. (4) yields orbits that resemble those around an optical vortex core [10]. A similar investigation with a different form of index has also been reported recently [29].

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