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Modeling and Tuning of Musical Percussive Beams

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Goal

Determine behavior of vibrational modes of a percussive beam when material is removed from a particular location. Model result qualitatively, and use results to design, cut, and test simple bars with specific overtone ratios.

Background

Sound is caused by vibration. When a metal bar vibrates, it does not do so randomly – it vibrates in discrete shapes or "modes," which occur at specific frequencies. These frequencies are determined by the material, geometry, and boundary conditions of the bar.

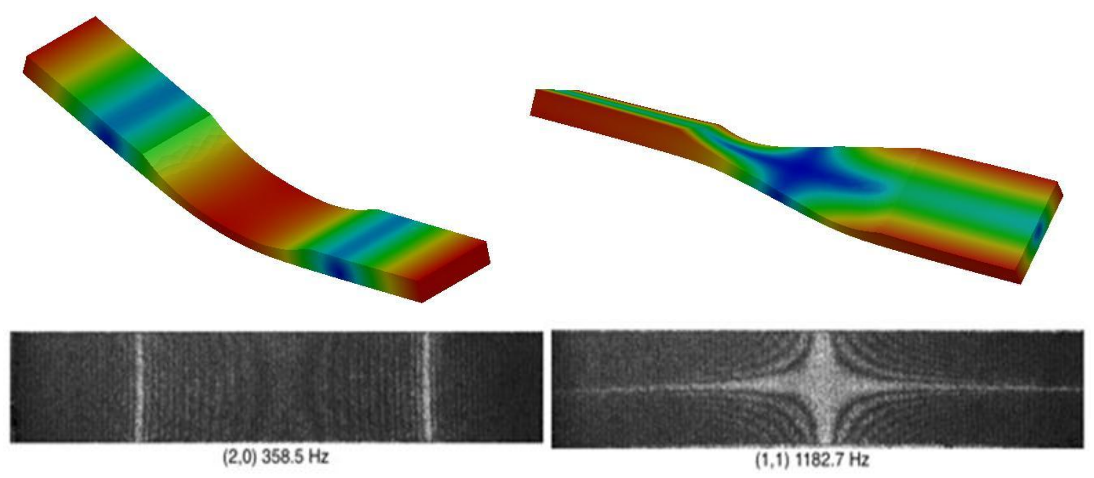

Figure 1: Two vibrational modes of a vibraphone bar, first modeled by Finite Element Analysis (FEA, top), then imaged with Electronic Speckle Pattern Interferometry (ESPI, bottom). The first number in parentheses is the number of vertical lines; the second is the number of horizontal lines. In FEA the blue regions correspond to nodes - places with no vibration. Nodes appear as white lines in ESPI.

When one strikes a metal bar with a mallet, it vibrates in many of these modes simultaneously, creating a sound layered with the respective frequencies of the vibrational modes. However, it is primarily the lowest pitch you hear – the (2,0) mode, also called the fundamental. The other pitches (primarily the lowest transverse modes) are called "overtones" and add character to the sound. Musically, it is often desirable to have the overtones occur at frequencies that are integer multiples of the fundamental frequency (giving them a "harmonic" relationship). Standard vibraphone bars consist of aluminum beams that are traditionally tuned with an arched undercut, for the purpose of aligning the musical overtones harmonically.

Figure 2. Cross-section of percussive bars showing an uncut bar (top) and a bar with a typical undercut on the bottom side (bottom).

Introduction

A number of papers [e.g. 1-4] detail the intricacies of tuning the overtones of percussive beams, enumerating on "practical tuning" techniques and even "optimal undercut" design. These publications use a range of methods, often involving a one-dimensional deformed beam model (see Theory) as well as a computational method such as finite element analysis (FEA) to achieve a given tuning ratio (i.e. 1:4:10 for the vibraphone). These models are useful and powerful tools. However, for those who seek a general qualitative model of how different cuts affect the lowest three transverse modes, this study elaborates on the work of Bork [1] in showing the effect of removing material from a given location and presents four general rules for the resultant behavior. These data were effectively used in conjunction with FEA to design simple and unique bars with ratios of 1:2:4, 1:3:5, and 1:4:10. These designs were recreated and verified with 2024-T4 aluminum flat bar in a machine shop.

Theory

The behavior of non-uniform percussive beams is difficult to describe mathematically, thus it is practical to model a uniform beam as a starting point. The behavior of a thin, isotropic beam of constant cross-section is modeled by Euler-Bernoulli theory:

$$EI\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} = 0 \qquad (1)$$

where E is the elastic modulus (Young's Modulus), I is the second moment of area of the cross-section, ρ is the density, and A is the cross-sectional area of the beam.

Using free-end boundary conditions, there is a standard one-dimensional result for the mode shapes with frequencies expressed:

$$f_n \sim \frac{h}{L^2}\sqrt{\frac{E}{\rho}}(2n+1)^2 \quad n=1,2,3,\ldots \qquad (2)$$

where h is the thickness of the bar, L is the length of the bar, and n is an integer that specifies mode number. It is useful to note that h is related to stiffness and ρ the mass. Thus, a small cut on the underside of the bar can be conceptualized as a reduction in both mass and stiffness. The resulting behavior of the mode frequency depends on competing factors: mass and stiffness. A reduction in mass will raise the frequency, whereas a reduction in stiffness will lower the frequency. Each mode has locations of varying amplitude and varying curvature. The general idea is that places with a higher degree of curvature will lend dominance to stiffness, while places with no curvature and a large degree of motion will cause mass to be the dominant effect. These effects are summarized in four qualitative rules for antinodes (places with the most motion) and nodes (places with no motion).

Qualitative Rules for Frequency Change Due to Removal of Material

Antinodes:

a) At the end of the bar, the curvature approaches zero, diminishing the effect of stiffness. Thus mass dominates, and the frequency increases slightly.

b) At non-end antinodes there are both mass and stiffness effects, but stiffness dominates (higher power dependence). Removal of material leads to a significant reduction in the frequency.

Nodes:

c) On nodes nearest the end there is no amplitude but there is slight curvature. This is enough for stiffness to have an effect, thus a removal of material leads to a moderate reduction in frequency.

d) On interior nodes there is no amplitude and no curvature, thus neither stiffness nor mass has any effect and there is no change in frequency.

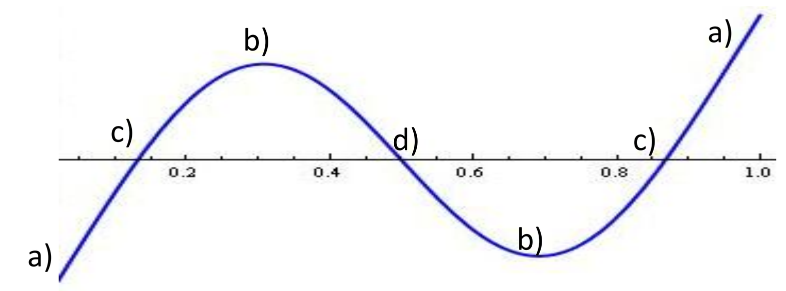

Figure 3: (3,0) mode shape. Labeled points correspond to nodes and antinodes described above.

Method/Procedure

Finite element analysis (FEA) was used to model the bar and compute the normal modes and their respective frequencies. This method was employed in order to obtain information about what happens to each mode as material is removed from particular locations. Cuts were modeled in symmetric pairs, reflected across the center of the bar, to maintain profile symmetry. Each cut had width 1/50th the total length of the bar and half the total depth of the bar. Additional models were constructed at certain points of interest for a range of depths.
The dimensions of the bar were inspired by a F4 stock vibraphone bar: 11.5 in. long, 2 inches wide, and half an inch deep.

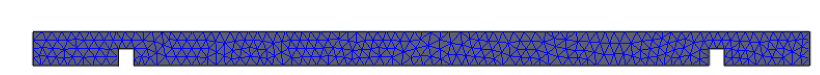

Figure 4: Example FEA model of material removal from an aluminum beam.

Ultimately the FEA results were used to custom-design bars with specific tuning ratios. The bars were cut out of aluminum bar stock. The acoustic frequencies and mode shapes were verified using a spectrum analyzer as well as an electronic speckle pattern interferometer. Once the custom-designed bars were cut, an audio recording was captured and pitch-normalized digitally for timbral comparisons between each bars' respective tuning ratios.

FEA Models and Analysis

The effect of removing material from specific locations was investigated in the same spirit as Bork [1], using FEA. The mode frequencies are plotted against cut location in figure 5 as a quotient of the original (uniform bar) frequency. This allows for the qualitative determination of where to make cuts to affect certain modes more than others, and which cuts will not affect certain ratios, &c.

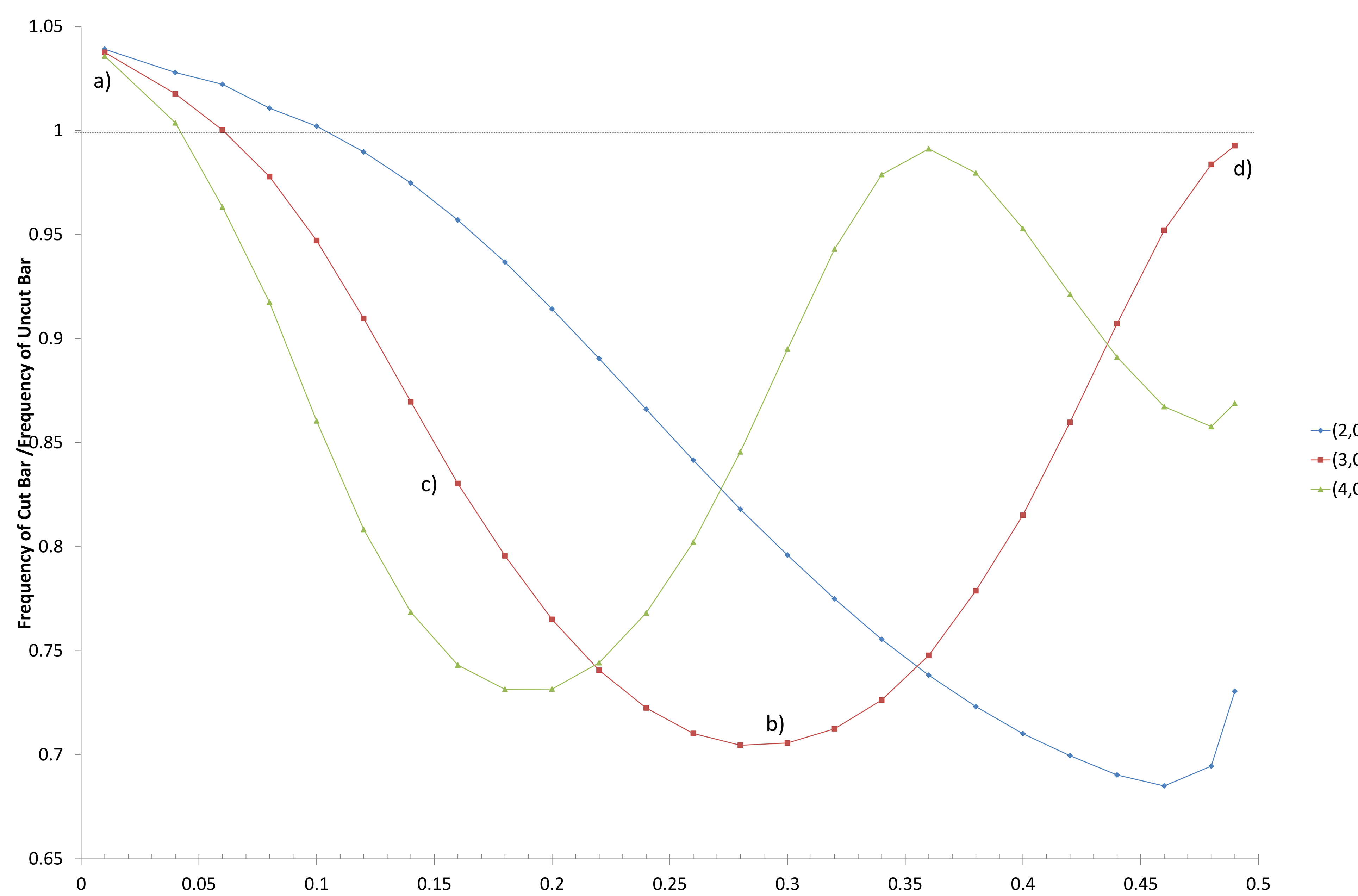

Figure 5: Effect on mode frequency of removing material from a percussive beam, as modeled by FEA. Material was removed in small cuts: 1/50th total length and ½ the total depth. Normalized to uniform bar frequencies. Plot shows half the length of the bar (other half is symmetric). The four qualitative rules for frequency behavior agree with the FEA results, as exemplified by the annotations on (3,0).

Places of intersection in figure 5 are of particular interest. They suggest that a cut at that location will preserve the frequency ratio of the intersecting modes, while either increasing or decreasing the other mode depending on whether it is above or below the intersection.

To gain an idea of how the intersections behave at different cut depths, additional models were made at these particular points of interest. Four depth charts were taken at x/L = 0.224, 0.275, 0.356 (shown in figure 6), and 0.44. In all but one depth study the two intersecting modes behaved similarly at only a small range of depths. However at x/L = 0.356 the (2,0) and (3,0) modes behaved identically from a cut depth 1/5 the total depth all the way to 4/5 the total depth of the bar. This proved to be an extremely useful piece of information in the design of bars with specific tuning ratios. I.e. if the ratios between (2,0) and (3,0) were as desired, (4,0) could be adjusted as necessary by adding a small cut at that location without affecting the other two. This idea was demonstrated effectively through the design of unique undercuts resulting in specific tuning ratios of 1:3:5, 1:2:4, and 1:4:10. While there was a certain element of trial-and error, the FEA results were useful in making a trial-and-error design relatively efficient.

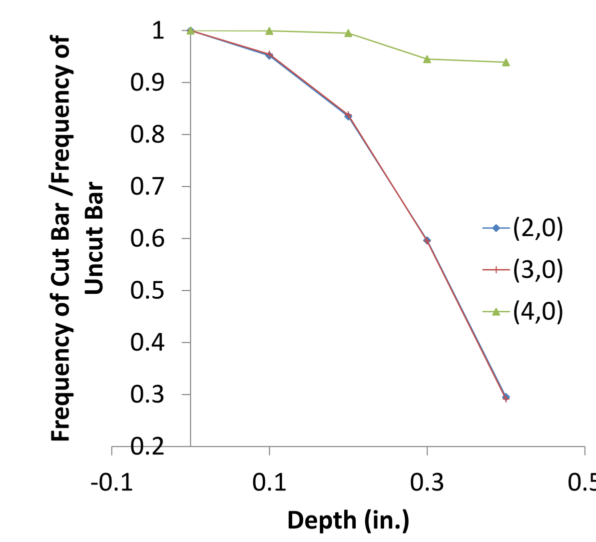

Figure 6: Example plot of depth analysis, showing mode frequency behavior of cuts centered at x/L = 0.356 from both ends with width 0.23" at varying depth. Note the exceptional agreement between (2,0) and (3,0) – this was atypical for the depth charts.

Construction

The computer-aided designs, based on simple rectangular slots, were recreated by cutting aluminum bars to specification using standard milling procedures. Using spectrum analysis and ESPI it was shown that the resulting frequencies were within 1% of the predicted frequencies.

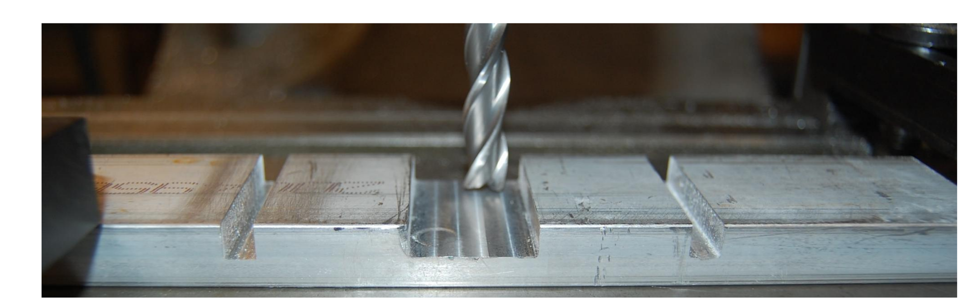

Figure 7: Custom bars were cut with an endmill on a milling machine.

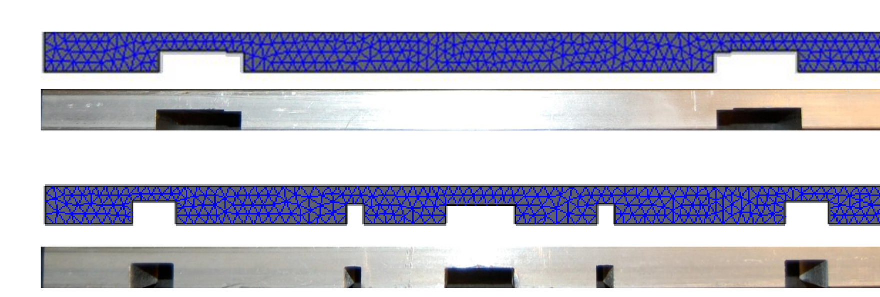

Figure 8: Computer model and cut bar with 1:2:4 overtone ratio (top pair) and 1:3:5 overtone ratio (bottom pair).

Conclusion

The effect of removing material from an aluminum bar was studied using FEA to model simple, symmetrical cuts. The FEA results made it possible to confirm a qualitative model of expected behavior, as well as guide the design of bars with custom overtone ratios. These designs were recreated with aluminum bars in a machine shop, and their acoustic properties verified with spectrum analysis and ESPI.

Ultimately, this study showed it is possible to make simple designs for harmonic percussive beams using qualitative physical analysis as a guide. This method was found to be accessible at the undergraduate level, and thus may present an alternative to more complex optimization techniques in the tuning of harmonic percussive beams.

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