By the early 20-th century, philosophers had developed a complex but purely formal semantics on logic that had already been established in their systems. There were two established methods to determine the logical validity of an argument. The first approach, following Euclid, became known as proof theory, while the second approach, following Aristotle, became known as model theory. It was well known that any argument in any first-order system which was proof-theoretically valid was also model-theoretically valid. But showing that a model-theoretically valid argument could be proven was a much harder problem. In 1929 Kurt Gödel published his Completeness Theorem, which finally established that for first-order systems, proof theory and model theory produced exactly the same results. This gave the logicians of the time further evidence that we were close to fully solving the problem of logical consequence. But it was only a two years later that Gödel established the now famous Incompleteness Theorems. Essentially, the first theorems says that, if our logical system is slightly more complex than a first-order system, capable of just basic arithmetic, then the system will always have statements which cannot be proved or disproved, while the second theorem says that there is no way to fix this problem. These incompleteness theorems are undoubtedly some of the most important results in philosophy of logic. Since then, modern logic have become an extremely wide and varied field (see 4, ch. 1). One of the most influential people in shaping modern logic was Alfred Tarski. In a short article called On the Concept of Logical Consequence, Tarski outlined what would eventually be incorporated into the standard model theory. A majority of the proof-theoretic and model-theoretic approaches that had been developed before are called syntactic systems. This meant that the logical systems divide the terms of a language into categories, treating each as something different. The idea behind the usual semantic system is that this is a system that takes the meaning of the terms in a language into account. This eventually became the standard system of logical consequence for much of the latter half of the 20-th century. Tarski's was not the first, a previous account of logical consequence that Tarski's school, the work of John W. Etchemendy, was the Concept of Logical Consequence, which offered a fundamental criticism of the Tarskian account of logical consequence. Model-theoretic semantics branched into roughly two schools of thought. The first is what Etchemendy calls interpretational semantics. This says that an argument is logically valid if it is impossible for the premises of the argument to be true, while the conclusion is false. It is quite difficult, however, to pin down exactly what should be counted as a possible situation. For example, is the argument “Roses are red and violets are blue. Therefore, roses are not (completely) red” logically valid? If roses were really blue, it is arguable metaphysiically impossible for roses to simultaneously be red. However, Etchemendy argues that Tarski's real goal was what he calls interpretational semantics. To illustrate interpretational semantics, he first outlines a similar but slightly simpler system called substitutional semantics first developed by Bolzano. In this system, we establish the logical validity of an argument in a similar way to Aristotelian. For example, the Aristotelian syllogism

All books are cold objects.
Therefore all books are cold.

Tarski, Gödel, and Etchemendy

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References