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Polarizing Majorana Fermions

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Background

Classically, a spin-1/2 fermion can interact electromagnetically via four methods: its charge, electric and magnetic dipole moments, and anapole moment. One can polarize a collection of these particles, such that their spins align, by applying an external field or current. The particle experiences a torque that aligns its spin with the direction of the applied field or current. One can see examples of classical electromagnetic interactions that result in polarization in the figure below.

EM moment	Distribution	Example	Interaction
Electric dipole			$\vec{N} = \vec{p}\times\vec{E}$ $U = -\vec{p}\cdot\vec{E}$
Magnetic dipole		spin	$\vec{N} = \vec{m}\times\vec{B}$ $U = -\vec{m}\cdot\vec{B}$
Anapole		Cs 55	$\vec{N} = \vec{a}\times\vec{J}$ $U = -\vec{a}\cdot\vec{J}$

Figure 1: Table showing classical EM configurations and how they interact with external currents or fields.

The Anapole Moment

- We can represent the anapole moment classically using a toroidal current distribution, pictured below:

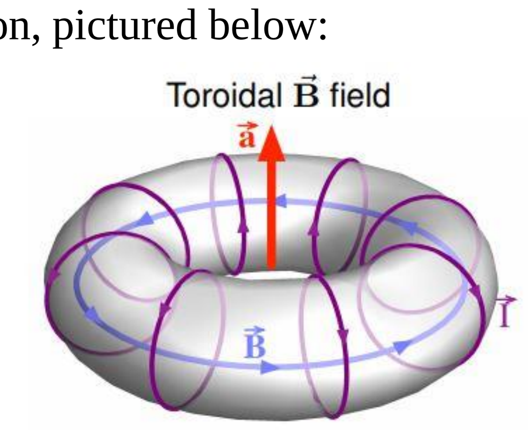


*Figure 2: A toroidal current distribution with anapole moment **a**. Current flows circularly around the torus. The magnetic field is confined to the interior.*

- The magnetic field is confined to the interior of the torus, the electric field is zero everywhere, and the dipole moments vanish.
- When interacting with a current, the anapole moment will align itself with the direction of the applied current.

The Majorana Fermion

- The Majorana fermion (MF) is a spin-1/2 particle that is its own antiparticle and has the following static EM properties:
  1. Electrically neutral
  2. Vanishing dipole moments
- Thus, it can only interact with a current via its anapole moment.
- We can then treat the MF as a point-like toroidal current distribution.

Research Objective

The objective of this research was to theoretically investigate methods by which one could polarize a collection of Majorana fermions, such that their spins align.

Acknowledgments and Future Work

- In the future, we aim to do more work with the MF-scalar current scenario. Through this, we hope to develop a way by which there will be a difference in energies between the two spin states.
- I would like to thank the Sherman Fairchild Foundation for funding this research.

Scattering Experiments

- We can analyze particle interactions through scattering experiments, where a projectile is deflected by a target particle at some angle θ.

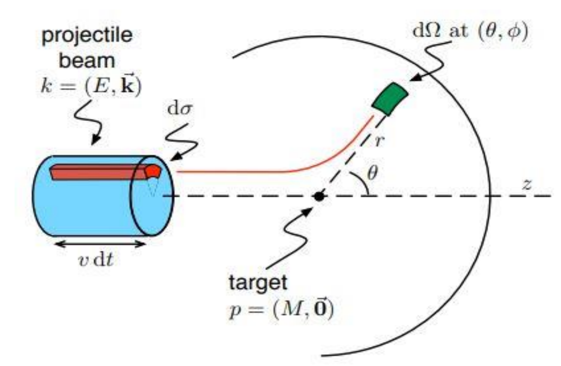
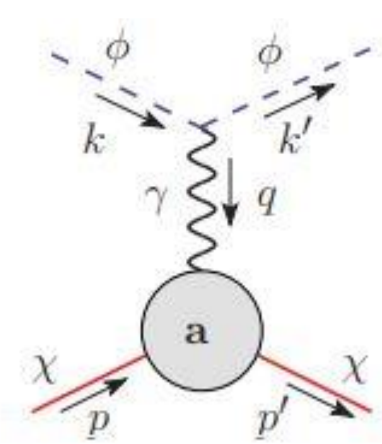


Figure 3: A diagram of a scattering experiment. For our purposes, the projectile beam will be some current, and the target will be the MF. When they interact, the current will be scattered at some angle θ.

- The scattering cross-section of the interaction is related to the probability that the projectile will be scattered at a specific angle, which can be measured experimentally.

Interaction with Scalar Current

- Figure 5 shows the Feynman diagram and corresponding expression for the scattering amplitude for a MF-scalar current interaction.



$$\mathcal{M} = ef_a\left(\frac{2k_\mu - q_\mu}{q^2}\right)\overline{u'}(p')(q^2\gamma^\mu - q^\mu\gamma^\alpha q_\alpha)\gamma^5 u(p)$$

Figure 5: The Feynman diagram and scattering amplitude equation for the MF-scalar current interaction.

- Working in the non-relativistic limit, we can make a few simplifying assumptions:
  1. Work in the rest frame of the MF
  2. Assume the mass of the MF is sufficiently large
- With these approximations, the scattering amplitude simplifies to:

$$\mathcal{M} = 4Mef_a\left\{-[(k+k')\cdot\mathbf{S}] + \frac{[(k+k')\cdot\mathbf{q}](\mathbf{q}\cdot\mathbf{S})}{|\mathbf{q}|^2}\right\}$$

- In the elastic scattering scenario, the scattering cross section is:

$$\frac{d\sigma}{d\Omega} = \left(\frac{1}{8\pi M}\right)^2\langle|\mathcal{M}|^2\rangle$$

- It was found that the scattering cross section is the same for the cases where the spin of the MF aligns and anti-aligns with the direction of the current:

$$\frac{d\sigma}{d\Omega}_{\downarrow\uparrow} = \frac{d\sigma}{d\Omega}_{\uparrow\downarrow} = \left(\frac{ef_a k}{4\pi}\right)^2\sin^2(\theta)$$

- This similarity in the scattering cross sections is due to the fact that we are working with a time-reversible process. This process does not take into account the energy difference associated with the different spin states of the MF, as pictured below.

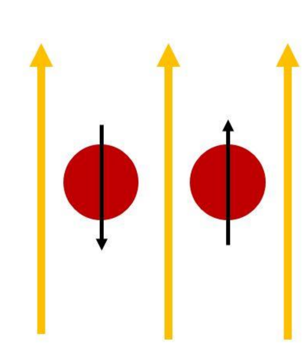


Figure 6: An illustration of how the MF with its spin aligned with an external current is in a lower energy state compared to a MF with its spin anti-aligned with the current. To get to the lower energy state, the MF must dump energy by some mechanism.

Theoretical Methods

- We can model particle interactions in quantum field theory by using Feynman diagrams.
- In these diagrams, each line and vertex corresponds to a mathematical function which can be used to calculate the scattering amplitude for a specific interaction.
- From the amplitude, one can determine the scattering cross-section for that interaction.

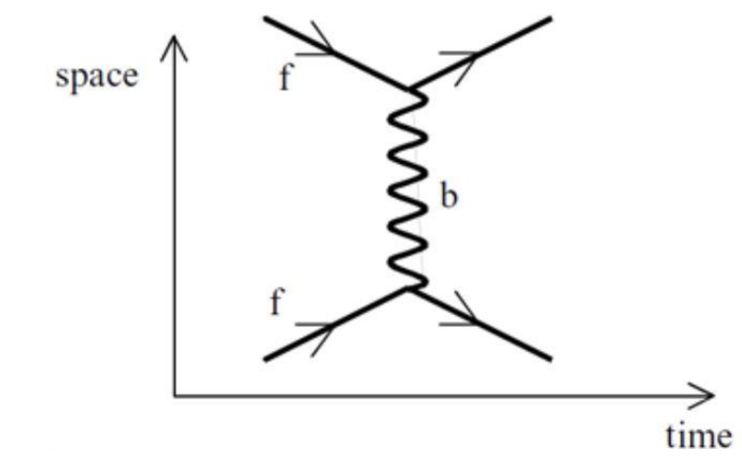
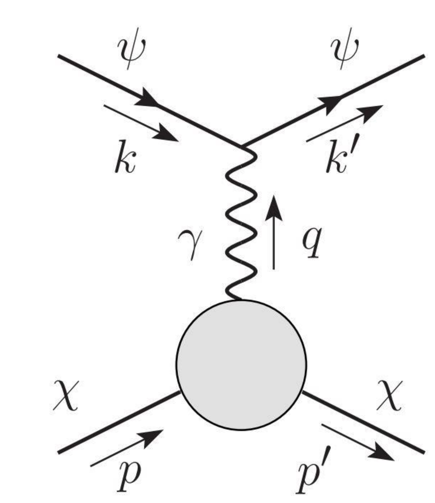


Figure 4: A general example of a Feynman diagram. Here, time is on the horizontal axis while space is on the vertical. Two particles come together with some initial momentum, interact through the exchange of a boson, and are then scattered with a new momentum.

Interaction with Fermion Current

- The Feynman diagram and scattering amplitude for this interaction are shown below.



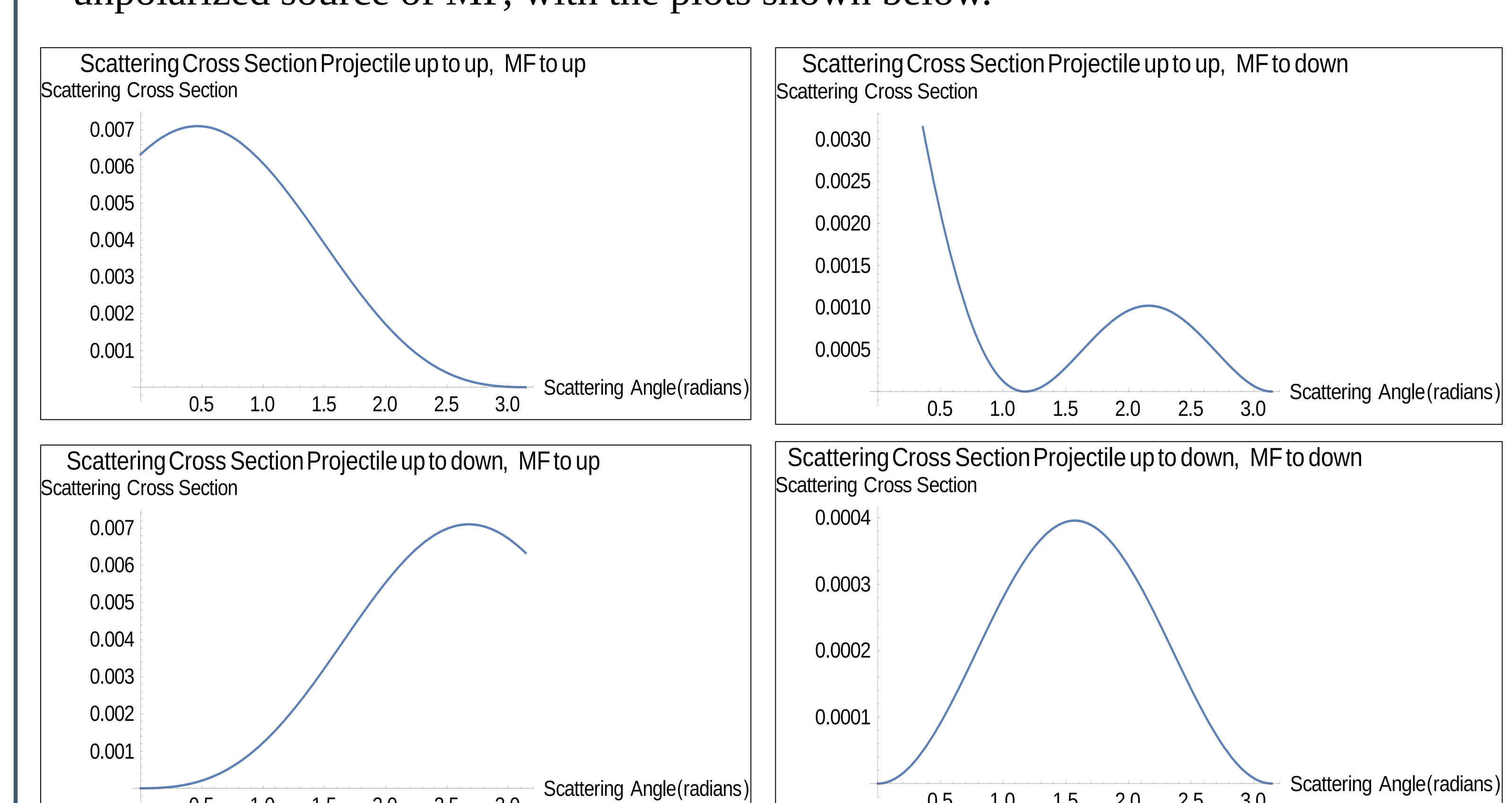
$$\mathcal{M} = \frac{if_a g_e}{q^2}\{\overline{u'}^{s_3}(p')(q^2\gamma^\mu - q^\mu\gamma^\alpha q_\alpha)\gamma^5 u^{s_1}(p)\}\{\overline{u'}^{s_4}(k')\gamma_\mu u^{s_2}(k)\}$$

Figure 7: The Feynman diagram and scattering amplitude equation for the MF-scalar current interaction.

- Using the same approximations as before, the above equation simplifies to:

$$\mathcal{M} = i2Mf_a g_e\left\{-\eta'^\dagger\eta[\mathbf{S}\cdot(k+k')] + \frac{\eta'^\dagger\eta}{|\mathbf{q}|^2}(\mathbf{q}\cdot\mathbf{S})[\mathbf{q}\cdot(k+k')] + i[\mathbf{S}\cdot(\mathbf{q}\times\mathbf{w})]\right\}$$

  - Here, η′ and η correspond to the final and initial spin states of the projectile particle
- The first to terms look similar to the scalar current interaction, but the third term is interesting because it involves the spins of both particles.
- We then calculated the scattering cross sections for four different scenarios, assuming an unpolarized source of MF, with the plots shown below.



- One can see that the greatest total cross section occurs for when the final state of the MF is aligned with the direction of the current (up).