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Continuum Model of Faceted Ice Crystal Growth in Cirrus Clouds in 1 Dimension

Ella Slattery

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Ella Slattery, Jake Price, Steven Neshyba

University of Puget Sound Department of Mathematics

Introduction

Cirrus clouds begin forming at 20,000 ft above sea level. Ice crystal growth in these clouds is not dendritic, like snowflakes, but rather **stable faceted growth**.

Figure 1. Cirrus clouds over Mount Rainier (left) and the Puget Sound (right).

- Ice crystals which form cirrus clouds reflect solar and infrared radiation from the sun.
- Roughening forms across ice crystal facets which affects cirrus cloud reflectivity [1].
- A mathematical model exhibiting stable faceted growth of these crystals may also exhibit roughening.
- Minimizing numerical instabilities in one dimension can allow for more efficient two-dimensional simulations.

Partial Differential Equation Models

Two Variable Model

$$\frac{\partial N_{tot}}{\partial t} = D\nabla^2 N_{qll} + \nu_{kin}\sigma_m \quad (1)$$

$$\frac{\partial N_{qll}}{\partial t} = D\nabla^2 N_{qll} + (\nu_{kin}\sigma_m)\frac{dN_{qll}}{dN_{tot}} \quad (2)$$

Diffusion term

The quasi-liquid water layer diffuses over the ice layer and periodically freezes.

Source term

Water vapor adds to the quasi-liquid water layer at a deposition rate that varies across the surface.

One Variable Models

$$\frac{\partial N_{tot}}{\partial t} = D\nabla^2[\bar{N} - N^*\sin(2\pi N_{tot})] + \nu_{kin}\sigma_m \quad (3)$$

$$\frac{\partial N_{qll}}{\partial t} = D\nabla^2 N_{qll} \pm 2\pi\nu_{kin}\sigma_m\sqrt{(N^*)^2 - (\bar{N} - N_{qll})^2} \quad (4)$$

Developing a Single Variable Model

- Equations (1) and (2) suggest that one variable models of this system can be developed if we consider the relationship between the quasi-liquid water layer and total thicknesses:

$$N_{qll}(N_{tot}) = \bar{N} - N^*\sin(2\pi N_{tot})$$

- One variable models were developed and are described by Equations (3) and (4).
- Equation (4) cannot accurately simulate any stable growth until the location of sign changes in the square root term are determined.

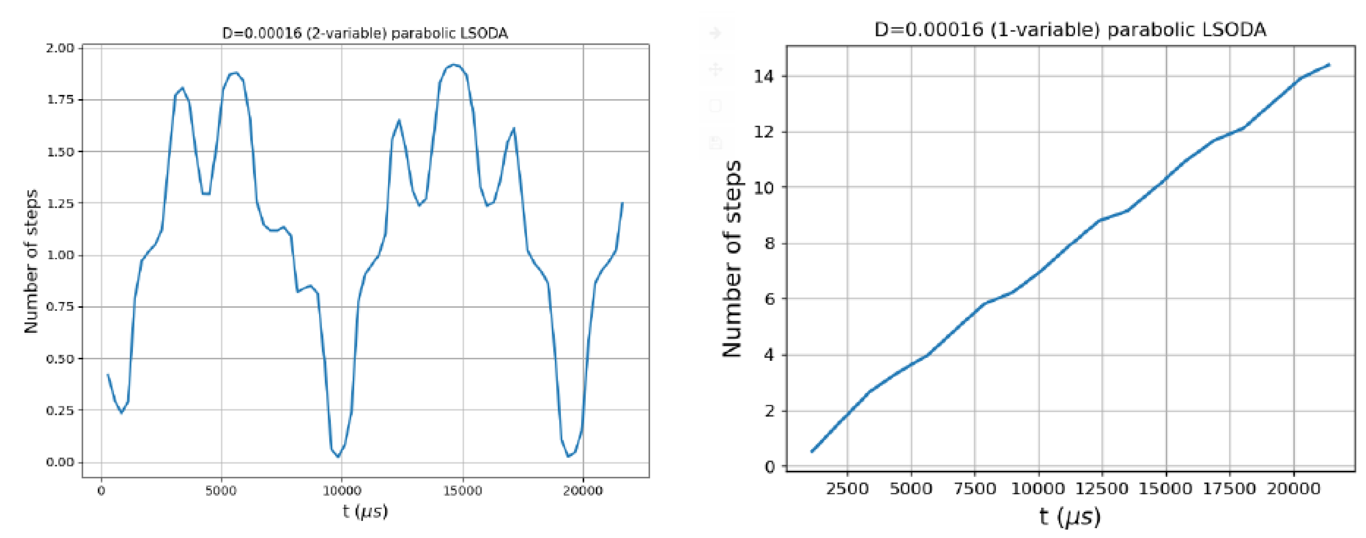

Figure 2. The number of steps achieved over time for the **two variable model (left)**, Equations (1) and (2), and the **one variable model (right)**, Equation (3).

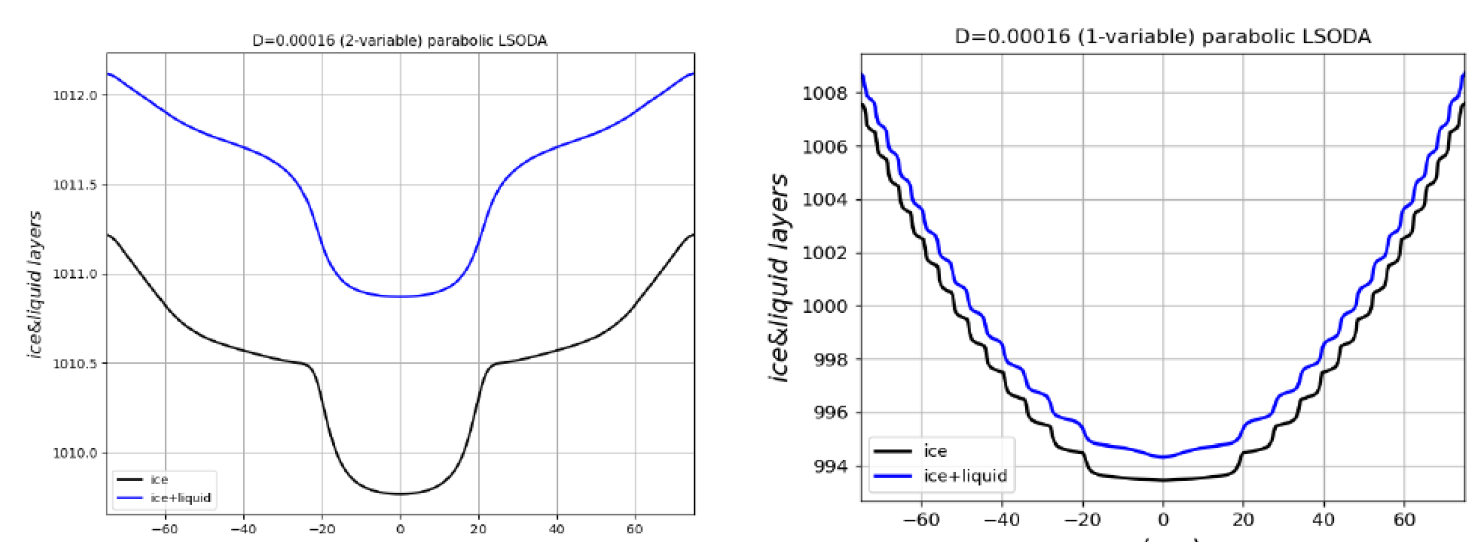

Figure 3. The quasi-liquid and ice layers of each model in Figure 2 respectively at the final timestep: the **total thickness of the layers (blue)** and the **thickness of the quasi-liquid water layer (black)**.

Acknowledgements

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Fourier Transform

$$\mathcal{F}[f(x,t)] = \int_0^{2\pi} f(x)e^{-ikx}dx$$

- The transform of the one variable model represented by **Equation (3)** resulted in a squared derivative, which cannot be transformed for our purposes.
- This prompted a **shift to the model described by Equation (4)**.

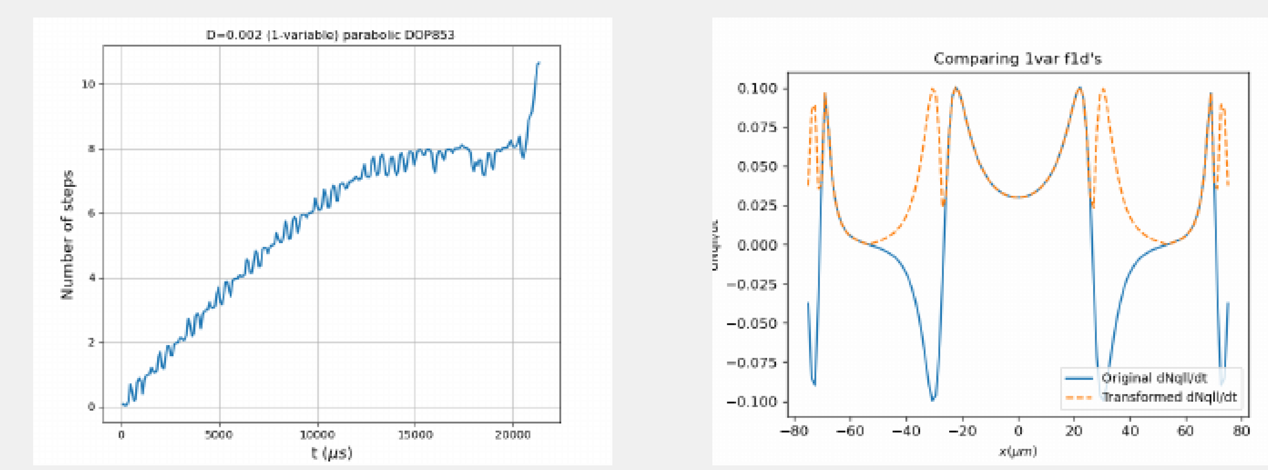

Figure 4. The **number of steps achieved over time (left)** for Equation (3) and the **derivatives calculated at the last timestep (right)** by the transformed Equation (4), in orange, compared to Equation (3), in blue.

Results

- The two variable system **tends towards stable limit cycles** over the one variable system.
- The two and one variable models **still exhibit numerical instabilities**.

Future Directions

- **Develop a theoretical explanation** for why the models are not identical.
- **Establish a condition** to determine the location of sign changes in Equation (4). Investigate the periodicity of related functions.
- **Continue work on the transform** in order to stabilize the model and run longer simulations. Determine whether a Fourier transform is appropriate for higher dimensional models.
- **Apply the model to 2 dimensions** and begin to describe roughening of the ice crystals along the prismatic facet.

[1] Neshyba, S., Adams, J.; Reed, K.; Rowe, P. M.; and Gladich, I. A quasi-liquid mediated continuum model of faceted ice dynamics, *Journal of Geophysical Research: Atmospheres* **2016**, 121 (23). 14,035–14,055. doi:10.1002/2016JD025458.