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[1]*Canberra Laboratory Manual* (Canberra Industries, Inc., 1977). *Experiments in Nuclear Science* (Ortec, Inc., 1976).
[2]G. I. Gleason, *Isotopic Neutron Source Experiments*, ORAU-102, Oak Ridge Associated Universities, 1967.
[3]Model TEL NU500 from Tel-Atomic, Inc., Model S441064 from Fisher Scientific, and Model 30191 from Central Scientific Co.
[4]Geiger–Muller tube from the Nucleus, also used in their Model 500.
[5]Ortec, Inc., Model 467.
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[7]A. C. Melissinos, *Experiments in Modern Physics* (Academic, New York, 1966), p. 187.
[8]Through a cylindrical water moderator of 30 cm diam × 50 cm height with the source in the center the surface strength is 0.5 mr/h of gamma activity and 0.7 mr/h of neutron activity as measured with a Victoreen Model 492 Survey Meter and a Victoreen Model 478 Portable Neutron Monitor.
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The operation of Maxwell's demon in a low entropy system

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The problem of Maxwell's sorting demon traditionally has been studied for the case in which the hot and cold regions differ very little in temperature. In this article a solution is presented for the case in which the temperature difference is great so that the total entropy is lower. Calculations indicate that in this case the demon must use a large number of photons to observe the proper kinds of particles. This causes an increase in entropy which more than offsets the decrease caused by an exchange of particles.

I. INTRODUCTION

The well-known sorting demon first proposed by Maxwell[1] remains one of the best examples of the wide range of applicability of the second law of thermodynamics. Maxwell was not able to reconcile the demon completely with the second law because he failed to take into account the energy (and hence the entropy) added to a system of particles by the act of observation. The necessary understanding of light quantization and black body radiation was not available at the time of Maxwell's death in 1879. Some relevant work concerning the relationship between quantum mechanics and the second law was done by Born and Green.[2,3] However, a more straightforward approach was taken by Brillouin,[4,5] who used only the "old" quantum theory of light quantization and black body radiation, along with classical thermodynamics. It is this approach to the problem of Maxwell's demon which is cited most often (although still infrequently) by thermodynamics texts and monographs.[6]

Brillouin's solution is valid over almost the entire range of classical thermodynamics. However, there are some particularly interesting examples with parameters lying outside of those allowed by Brillouin's simplifying assumptions. This article will consider some of those examples. Whenever necessary the most generous concessions will be allowed the demon. In spite of these concessions, the demon (not surprisingly) still will not be able to violate the second law.

II. THEORY

Brillouin considered two gas-filled regions A and B separated by a movable partition. The partition is controlled by the demon, who hopes to decrease the total entropy of the system by allowing only relatively fast particles to pass from the cold to the hot region. It is assumed that $T_B > T_A$, with $T_B - T_A = \Delta T$. Also, a temperature T is defined so that

$$T_A = T - \tfrac{1}{2}\Delta T \quad (1)$$

and

$$T_B = T + \tfrac{1}{2}\Delta T. \quad (2)$$

In each region j the equipartition theorem dictates that the mean energy of a particle is

$$\overline{E}_j = \tfrac{3}{2} kT_j\,. \quad (3)$$

Therefore the demon should select a particle from A with energy $\frac{3}{2} kT\,(1+\epsilon_1)$ and allow it to pass into B. Similarly, a particle with energy $\frac{3}{2} kT\,(1-\epsilon_2)$ will be allowed to pass from B into A.

In order to see those particles, one must distinguish photons scattered from them from the black body radiation.

This requires that the energy of the photon used to observe the particle is $h\nu > kT$. The entropy increase due to the act of observation is then

$$\Delta S_d = h\nu/T = kb, \quad (4)$$

where $b = h\nu/kT > 1$. Since (at least) two photons must be used in the process described above, the net entropy increase is

$$\Delta S_d = 2kb. \quad (5)$$

The entropy decrease effected by the demon is calculated using the energy transfer

$$Q = \tfrac{3}{2}kT(\epsilon_1 + \epsilon_2). \quad (6)$$

The resulting entropy decrease is

$$\Delta S_i = (Q/T_B) - (Q/T_A) \cong -Q(\Delta T)/T^2 \quad (7)$$

or

$$\Delta S_i \cong -\tfrac{3}{2}k(\epsilon_1 + \epsilon_2)(\Delta T/T). \quad (8)$$

Now Brillouin presumes that both ϵ_1 and ϵ_2 are small and that $\Delta T \ll T$, which means that

$$\Delta S_i = -\tfrac{3}{2}k\eta, \quad (9)$$

with $\eta \ll 1$. Therefore the net entropy change is

$$\Delta S_d + S_i \cong k(2b - \tfrac{3}{2}\eta) > 0 \quad (10)$$

and the second law is validated.

The example just presented is not valid universally, however. Crucial to this argument is the assumption in Eq. (7) that $T_A T_B \cong T^2$. The exact expression is

$$T_A T_B = T^2 - (\Delta T)^2/4. \quad (11)$$

The resulting entropy decrease due to energy exchange is now

$$\Delta S_i = -Q(\Delta T)/[T^2 - (\Delta T)^2/4]. \quad (12)$$

ΔS_i is not necessarily negligible any more. In the limit as ΔT approaches T (the most favorable case from the demon's point of view),

$$\Delta S_i = -\tfrac{3}{4}k(\epsilon_1 + \epsilon_2)[(T_B^2 - T_A^2)/(T_A T_B)], \quad (13)$$

or using Eq. (6),

$$\Delta S_i \cong -\tfrac{3}{4}k(\epsilon_1 + \epsilon_2)(T_B/T_A). \quad (14)$$

A comparison of Eqs. (5) and (14) reveals where the difficulty lies. By definition $\epsilon_2 < 1$, but in principle ϵ_1 may be somewhat greater than unity. At first glance this would seem to be a violation of the second law, for while b must be greater than unity in order to distinguish a particle from the black body radiation, there is no clear justification for it to be greater than ϵ_1.

The resolution of this problem lies in the fact that a single photon will not be sufficient to find a particle with the desired velocity. Consider how the values of ϵ_1 relate to the physical situation. In order to obtain a relatively high value for ϵ_1 (even to obtain a value approaching unity), the demon must select a particle with exceptionally high speed compared with the mean.

$$\bar{v}_A = \sqrt{8kT_A/\pi m}, \quad (15)$$

where m is the mass of a single particle. Particles with this desired characteristic will be found only far out on the "tail" of the Maxwell speed distribution. The fraction F of those particles with a speed greater than a given speed v_2 for a classical gas at temperature T_A is

$$F = \int_{v_2}^{\infty} f(v)dv, \quad (16)$$

where $f(v)$ is the normalized Maxwell function

$$f(v) = 4(m/2\pi kT_A)^{3/2}v^2 \exp(-mv^2/2kT_A). \quad (17)$$

This means that the number n_1 of photons normally re-

Table I. Calculated values of n_1, n_2, ϵ_1, and ϵ_2 as a function of temperatures and selected speeds for nitrogen gas. Temperatures are in K and speeds in m/s.

T_A	T_B	$\bar{v}_A$	$\bar{v}_B$	v_1	v_2	n_1	n_2	ϵ_1	ϵ_2
300	320	475	490	300	600	3.9	5.3	0.31	0.67
300	320	475	490	100	800	16	113	1.3	0.96
300	320	475	490	100	1200	1100	113	4.3	0.96
200	220	388	407	300	500	4.2	3.4	0.34	0.52
200	220	388	407	200	600	9.4	9.3	0.94	0.78
200	220	388	407	200	1000	1500	9.3	4.4	0.78
100	120	274	300	200	400	7.0	4.3	0.64	0.59
100	120	274	300	100	500	27	27	1.6	0.90
100	120	274	300	100	1000	7.2×10^6	27	9.3	0.90
600	620	672	683	600	800	3.3	2.4	0.19	0.33
600	620	672	683	400	1000	7.7	5.9	0.85	0.70
600	620	672	683	400	2000	2.4×10^4	5.9	6.4	0.70
2000	2100	1226	1256	1000	2000	13	2.9	1.2	0.45
2000	2100	1226	1256	1000	3000	670	2.9	4.0	0.45
300	500	475	612	300	800	16	9.5	0.81	0.75
300	500	475	612	300	1200	1100	7.5	3.1	0.75
300	1000	475	867	300	1200	1100	25	1.5	0.84
300	1000	475	867	200	2000	2.4×10^9	79	6.0	0.93
200	500	388	613	200	800	82	29	1.1	0.87
200	500	388	613	200	1250	6.0×10^4	29	3.7	0.87
100	500	274	613	100	800	1.6×10^4	220	1.4	0.96
100	500	274	613	100	1000	7.2×10^6	220	2.8	0.96

Table II. Calculated values of n_1, n_2, ϵ_1, and ϵ_2 as a function of temperatures and selected speeds for hydrogen gas. Temperatures are in K and speeds in m/s.

T_A	T_B	$\bar{v}_A$	$\bar{v}_B$	v_1	v_2	n_1	n_2	ϵ_1	ϵ_2
300	320	1777	1835	1500	2100	3.2	2.8	0.15	0.41
300	320	1777	1835	1000	2500	6.0	7.1	0.63	0.74
300	320	1777	1835	1000	3000	16	7.1	1.3	0.74
300	320	1777	1835	1000	3500	53	7.1	2.2	0.74
200	220	1451	1521	1200	2000	5.5	3.0	0.54	0.45
200	220	1451	1521	1000	3000	84	4.5	2.5	0.62
30	50	562	725	300	1000	23	15	1.0	0.82
30	50	562	725	300	1500	2800	15	3.5	0.82
600	620	2513	2554	2000	3000	3.3	3.0	0.19	0.47
600	620	2513	2554	2000	5000	58	3.0	2.3	0.47
2000	2100	4587	4701	4000	6000	4.5	2.5	0.42	0.37
2000	2100	4587	4701	4000	10000	150	2.5	2.9	0.37
300	1000	1777	3244	1500	3500	53	11	0.52	0.72
300	1000	1777	3244	1000	5000	7400	34	2.1	0.88
200	1000	1451	3244	1400	3500	550	13	0.65	0.74
200	1000	1451	3244	1400	5000	1.1×10^6	13	2.4	0.74
100	1000	1026	3244	1000	3300	1.6×10^5	34	0.60	0.85
100	1000	1026	3244	1000	3500	8.4×10^5	34	0.80	0.85

quired to find a particle with speed greater than v_2 will be

$$n_1 = \frac{1}{F} = \left(\int_{v_2}^{\infty} f(v)dv\right)^{-1}. \quad (18)$$

Similarly the demon must select an appropriately slow particle for transfer from B to A. Clearly the number of photons needed to find such a particle (with speed less than v_1) will be

$$n_2 = \left(\int_0^{v_1} f(v)dv\right)^{-1}. \quad (19)$$

Note that the temperature T_B must be used in $f(v)$ in Eq. (19). Also, it is clear that v_1 must not be more than $\bar{v}_A$ and v_2 must not be less than $\bar{v}_B$. With these constraints the net entropy change will be

$$\Delta S_d + \Delta S_i \geqslant k(n_1 b + n_2 b - 2\epsilon_1 - 2\epsilon_2), \quad (20)$$

where the inequality is due to the nature of the approximations made to obtain Eqs. (13) and (14). The sum of n_1 and n_2 must be sufficiently large in order for (20) to satisfy the second law, i.e., so that the right-hand side of (20) is positive.

III. RESULTS

The integrals in Eqs. (18) and (19) were performed on a computer using Newtonian quadrature. The integral in Eq. (18) was calculated using limits v_2 to $10v_2$. The exponential factor in the Maxwell function ensures that these limits will be sufficient to yield a very accurate result.

Tables I and II show some results for the typical gases nitrogen and hydrogen. Since ϵ_2 is restricted to values $0 < \epsilon_2 < 1$, it is useful to concentrate on ϵ_1. As was noted in Sec. II, higher values of ϵ_1 represent a greater entropy de-

Table III. Calculated values of n_1, n_2, ϵ_1, and ϵ_2 as a function of temperatures and selected speeds for a dilute gas of electrons. Temperatures are in K and speeds in m/s.

T_A	T_B	$\bar{v}_A$	$\bar{v}_B$	v_1	v_2	n_1	n_2	ϵ_1	ϵ_2
300	320	1.08×10^5	1.12×10^5	1.0×10^5	1.2×10^5	2.7	2.3	0.01	0.30
300	320	1.08×10^5	1.12×10^5	1.0×10^5	2.0×10^5	31	2.3	1.8	0.30
300	320	1.08×10^5	1.12×10^5	1.0×10^5	3.0×10^5	5600	2.3	5.3	0.30
100	120	6.2×10^4	6.8×10^4	5.0×10^4	1.0×10^5	11	3.5	0.98	0.50
100	120	6.2×10^4	6.8×10^4	5.0×10^4	2.0×10^5	1.4×10^5	3.5	6.9	0.50
1	2	6200	8800	5000	1.5×10^4	515	6.5	2.3	0.64
1	2	6200	8800	1900	2.0×10^4	1.4×10^5	6.5	4.8	0.64
0.1	1	1974	6242	1900	6500	3.0×10^5	35	0.67	0.86
0.1	1	1974	6242	1900	1.0×10^4	9.5×10^{13}	35	3.0	0.86
1.0×10^{-6}	2.0×10^{-6}	6.2	8.8	6	10	12	4.1	0.45	0.48
1.0×10^{-6}	2.0×10^{-6}	6.2	8.8	6	20	1.4×10^5	4.1	4.8	0.48
1000	1500	2.0×10^5	2.4×10^5	1.9×10^5	2.5×10^5	4.0	3.0	0.09	0.37
1000	1500	2.0×10^5	2.4×10^5	1.9×10^5	3.0×10^5	8.6	3.0	0.57	0.37
1000	1500	2.0×10^5	2.4×10^5	1.9×10^5	4.0×10^5	69	3.0	1.8	0.37
1000	2000	1.0×10^5	2.8×10^5	1.9×10^5	3.0×10^5	8.6	4.1	0.31	0.48
1000	2000	2.0×10^5	2.8×10^5	1.9×10^5	4.0×10^5	69	4.1	1.3	0.48

crease associated with the exchange of particles between A and B. The calculations presented in Tables I and II suggest strongly that there is no adjustment of parameters which will result in a favorable outcome for the demon, i.e., a net decrease in entropy. The best strategy for increasing ϵ_1, namely to increase v_2, is precisely the thing which causes n_1 to increase most quickly. In all cases examined it is clear that n_1 increases much more rapidly than ϵ_1. This should not be unexpected considering the nature of the statistical distribution of speeds.

In order to determine whether this result is peculiar to typical gas molecules, calculations were also performed for a gas of electrons. It is assumed that the gas is dilute enough so that Coulomb forces do not adversely affect the work of the demon. This assumption also makes it reasonable to believe that classical statistics will be valid over the wide range of temperatures considered. Again the results (shown in Table III) indicate that any attempt to increase ϵ_1 results in a much greater increase in n_1. This is true for a wide range of temperatures T_A and T_B.

While these results are by no means exhaustive, they provide further insight into the problems faced by Maxwell's entropy reducing demon. Indeed it is clear that these entropy reductions can only be slight in comparison with the entropy added through the required process of observation. It is interesting to note what a crucial role this process of observation (normally important in modern quantum theory) plays in dealing with particles which obey classical statistics. This is perhaps one of the best examples of such an overlap between classical and quantum concepts.

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The gravitational Poynting vector and energy transfer

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We define, in analogy to electrodynamics, the gravitational Poynting vector and show that it provides a mechanism for the transfer of gravitational energy to a system of falling objects.

By considering a special system (i.e., a point mass moving in the gravitational field of a line mass) it is easily demonstrated on relativistic grounds that the Newtonian gravitational interaction between masses appears in two distinct though interrelated forms.[1] In addition to the well-known static interaction $\mathbf{F} = m\mathbf{g}$, where for instance $g = -GM/r^2$ for a point mass M (G is the gravitational constant) there is a "gravinetic" interaction between moving masses $\mathbf{F} = m\mathbf{v}\times\mathbf{b}$, where $\mathbf{v}$ is the velocity of the mass m, moving through the "gravinetic" field $\mathbf{b}$. For a point mass M moving with velocity $\mathbf{V}$ this field takes the form $\mathbf{b} = -(G/c^2)(M\mathbf{V}\times\hat{\mathbf{r}}/r^2)$, where $\hat{\mathbf{r}}$ is a unit vector in the direction of $\mathbf{r}$, which points from the moving mass M to the field point.

The existence of a velocity-dependent gravitational interaction is well known from the general theory of relativity: the off-diagonal elements of the gravitational field tensor are indeed velocity dependent, but are usually dismissed as insignificant in the weak field approximation.[2] Furthermore, the fact that gravinetic forces appear as a consequence of the special theory of relativity, although briefly mentioned in some textbooks on electrodynamics,[3] has not penetrated into the basic literature on mechanics and gravity. This is an unfortunate omission, particularly since it could serve as an accessible introduction to some features of General Theory. In this paper we would like to point out that the "gravitational Poynting vector," formed in analogy to the electromagnetic one, *is* significant, and

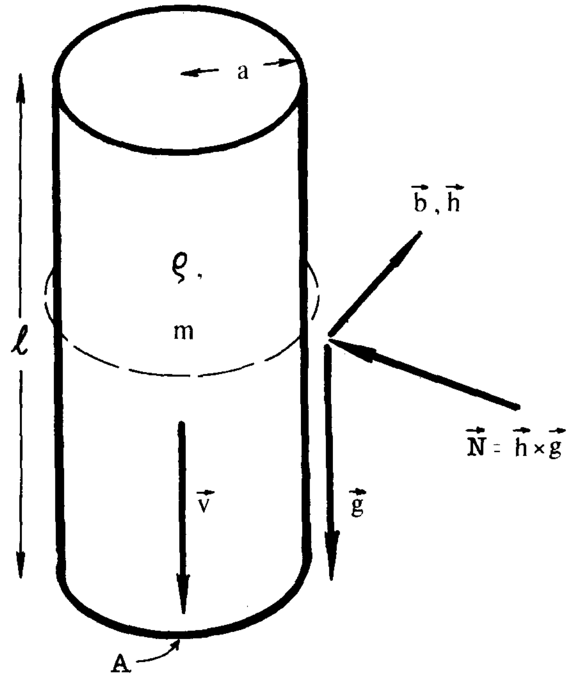

Fig. 1. The cylinder, mass m, falling with velocity $\mathbf{v}$ in the gravitational field $\mathbf{g}$.

