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Quantum Conundrums: Putting quantum mechanics to the test with Bell's Inequality

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Putting quantum mechanics to the test with Bell's Inequalities

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In 1935, Einstein, along with Boris Podolsky and Nathan Rosen, introduced the EPR paradox which stated that either quantum mechanics is incomplete with some sort of hidden variable present but unknown, or it violates the fundamental relationship of causality. Despite Einstein's best efforts, this paradox never particularly tore down the foundations of quantum mechanics, but it did remain unresolved for many years until John Bell's 1964 introduction of "Bell's Inequality." Bell proposed an experiment involving pairs of entangled particles emitted from a single source and showed that the correlation between measurements on each independent particle (particle spin, polarity, etc.) is different using the quantum mechanical interpretation as compared to any "hidden variable theory." In 2001, Dietrich Dehlinger and Morgan Mitchell performed a thorough test feasible on the undergraduate scale for this advanced inequality, and for the purposes of our research, we used this as a basic model for our experimental set-up. To put the debate to rest and witness the rarely seen effects of quantum mechanics first-hand, my research here at the University of Puget Sound used the polarity of entangled photon pairs produced by spontaneous parametric down conversion to demonstrate Bell's inequality and the legitimacy of quantum mechanics.

Experimental Set-up

To understand Bell's Inequality, one must first understand the specific context of our experiment. Initially, 402 nm photons just within the visible blue range are produced by a diode laser. To ensure uniform polarization and wavelength, the beam passes through a polarizer and a blue filter. A pair of lenses collimates the beam into one single point and a rotatable quartz plate introduces a phase shift to the incoming light. Finally, the photons pass through a pair of birefringent beta barium borate crystals to undergo what is known as spontaneous parametric down conversion. In this process, the input or "pump" photon is converted into two separate photons, the "signal" and "idler" photons. Coming from a single parent photon, certain characteristics are interrelated and these photons are considered "entangled." For instance, the energies of the two downconverted photons must add up to that of the pump photon and the signal and idler photon polarizations are identical.

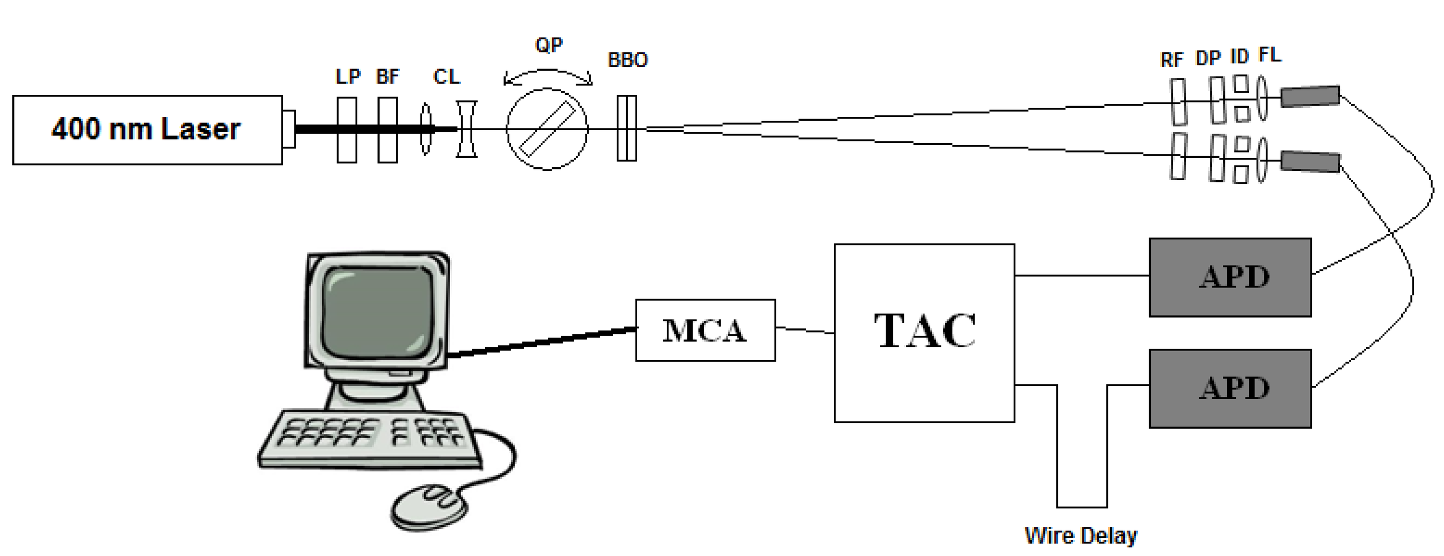

Figure #1: Theoretical diagram of experimental set-up. (LP=Laser Polarizer, BF=Blue Filter, CL=Collimating Lenses, QP=Quartz Plate, BBO=BBO Crystal, RF=Red Filter, DP=Detector Polarizer, ID=Iris Diaphragm, FL=Focusing Lens, APD=Avalanche Photodiode, TAC=Time-to-Amplitude Converter, MCA=Multichannel Analyzer)

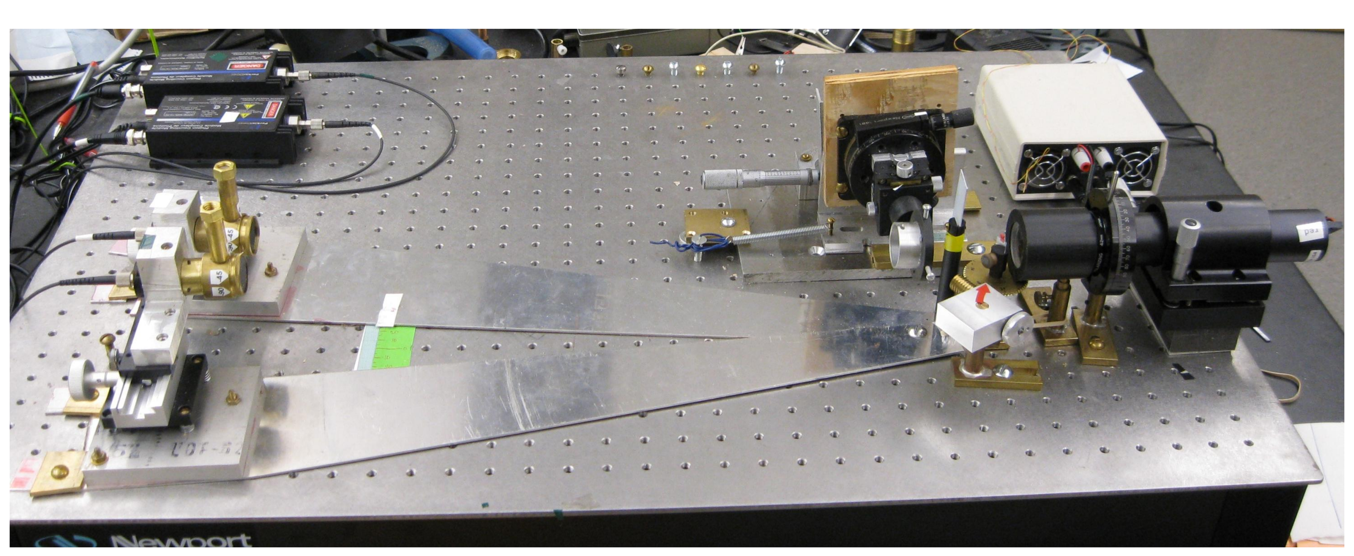

Figure #2: Actual Experimental Set-up

In this experiment, we only consider the case of signal and idler photons of half the energy of the input (804 nm wavelengths) and output polarizations perpendicular to that of the pump photons. In accordance with the conservation of momentum, our downconverted photons veer off at an angle of ±3° with relation to the original beam. At the end of these paths, photons are passed through red filters and focused onto two avalanche photo diodes (APD) to detect coincidences in these photons to ensure the consideration of only downconverted light. The output of the APDs is then passed through a wire delay and sent to a time-to-amplitude converter (TAC). A multichannel analyzer interprets the output of the TAC and finally displays a graph showing registered photon detections versus the time delay between the two detections. Whatever is located at the time of our wire delay is therefore downconverted pairs. To measure the correlation between the two entangled photons, rotatable linear polarizers are placed in front of the detectors and the variance of the coincidence counts based on the individual polarizer angles is interpreted mathematically in the following manner.

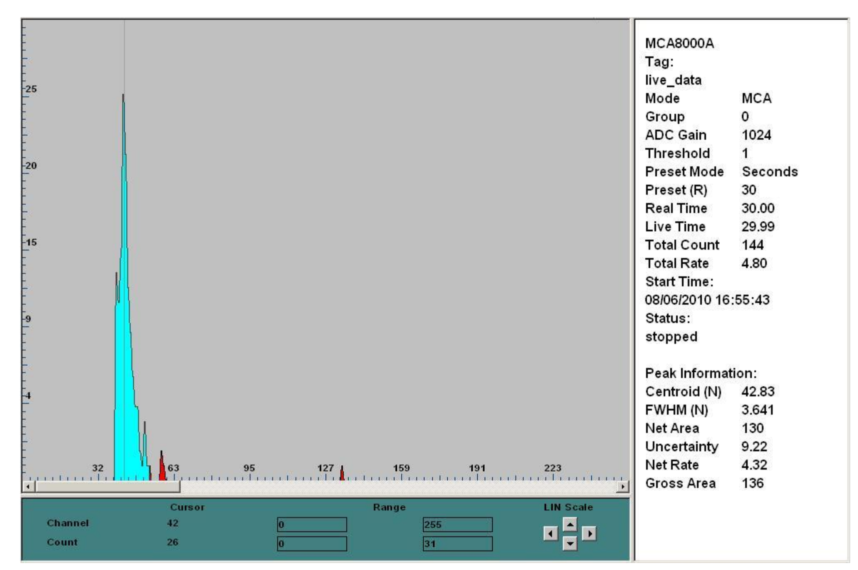

Figure #3: Sample MCA Output Graph

Theory

In the "hidden variable" interpretation, the polarization of a photon is at some specific angle ϕ, and when the photon meets a polarizer set to an angle ω, it registers as vertical with respect to that polarizer simply if it is closer to ω rather than perpendicular, making the probability of vertical detection as follows.

$$P_V^{(HVT)}(\omega,\varphi) = \begin{cases} 1 & |\omega-\varphi| \le 45° \\ 1 & |\omega-\varphi| > 135° \\ 0 & \text{otherwise} \end{cases}$$

So, in our context, the probability of detecting a pair of downconverted photons would ignore any previous actions involving polarization or phase shift and simply be the product of the two probabilities of vertical detection through the individual polarizer angles on each leg, α and β respectively. This eliminates the photon polarization term and leaves us with the linear expression as shown here, dependent only on the difference between α and β.

$$P_{VV}^{(HVT)}(\alpha,\beta) = \frac{1}{2} - \frac{|\beta-\alpha|}{\pi}$$

However, from the quantum mechanical viewpoint, the polarization of a photon is seen as a combination of vertically and horizontally polarized quantum states. Passing through the initial laser polarizer set at an angle and birefringent quartz plate with a phase shift of , the pump photons can be described in the quantum state

$$|\psi_{\text{pump}}\rangle = \cos\theta_l\,|V\rangle_p + \exp[i\Phi_l]\sin\theta_l\,|H\rangle_p$$

As mentioned in the experimental set-up, after passing through the BBO crystal, our Type-I down conversion produces signal and idler photons with polarizations perpendicular to the pump photon polarization. Due to the birefringence of the BBO crystal (different indices of refraction for different polarizations), another phase shift is taken into account with the total denoted as *Φ*, producing the following quantum state.

$$|\psi_{\text{DC}}\rangle = \cos\theta_l\,|H\rangle_s|H\rangle_i + \exp[i\Phi]\sin\theta_l\,|V\rangle_s|V\rangle_i$$

In the same general fashion of the laser polarizer, the detector polarizers, set at angles α and β respectively, pass photons in the quantum states

$$|V_\alpha\rangle = \cos\alpha|V\rangle - \sin\alpha\,|H\rangle \quad \text{and} \quad |V_\beta\rangle = \cos\beta|V\rangle - \sin\beta|H\rangle.$$

Therefore, by projecting the downconverted quantum state onto these polarizer quantum states, we can calculate the quantum mechanical probability of coincidence detection by multiplying this projection with its complex conjugate.

$$P_{VV}^{(QM)}(\alpha,\beta) = \left|\langle V_\alpha, V_\beta|\psi_{\text{DC}}\rangle\right|^2 = |\sin\alpha\sin\beta\cos\theta_l + \exp[i\Phi]\cos\alpha\cos\beta\sin\theta_l|^2$$

$$P_{VV}^{(QM)}(\alpha,\beta) = \sin^2\alpha\sin^2\beta\cos^2\theta_l + \cos^2\alpha\cos^2\beta\sin^2\theta_l + \frac{1}{4}\sin2\alpha\sin2\beta\sin2\theta_l\cos\Phi$$

This equation simplifies when the total phase shift, *Φ*, is normalized to zero and the laser polarizer angle, , is set to 45 degrees to equalize the horizontal and vertical quantum states, producing a final probability of

$$P_{VV}^{(QM)}(\alpha,\beta) = \frac{1}{2}\cos^2(\beta-\alpha).$$

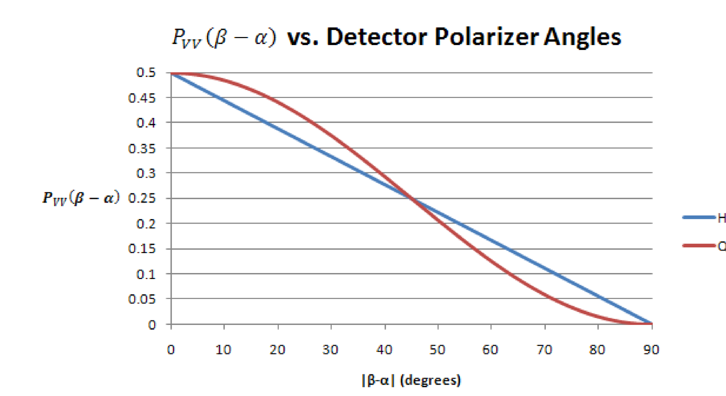

Figure #3: Graph of Detection Probability for both interpretations varying the angle difference.

As you can see in Figure #3, both interpretations follow the same general pattern, partially explaining why there has been such difficulty in discerning which one is valid, but noticeable differences can be seen at certain angles, specifically 22.5° and 67.5°. What Bell's Inequality seeks to do is to exploit these small but noticeable differences in numerical form and experimentally show whether nature prefers one theory or the other. This is done through a type of correlation measurement of detection on the two detectors by adding the probability of detection agreement (HH or VV) and subtracting the probability of disagreement (HV or VH).

$$E(\alpha,\beta) = P_{VV}(\alpha,\beta) + P_{HH}(\alpha,\beta) - P_{HV}(\alpha,\beta) - P_{VH}(\alpha,\beta)$$

Experimentally, this statistic is calculated by taking coincidence counts using α and β as well as the angles perpendicular to them for horizontal detection in the following manner.

$$E(\alpha,\beta) = \frac{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) - N(\alpha_\perp,\beta) - N(\alpha,\beta_\perp)}{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) + N(\alpha_\perp,\beta) + N(\alpha,\beta_\perp)}$$

Finally, using four different polarizer angles, four of these *E* factors are added together to produce $S = E(a,b) - E(a,b') + E(a',b) + E(a',b')$. The inequality implies that any hidden variable theory can only produce a value of S less than 2, whereas the quantum mechanical theory can produce values up to $2\sqrt{2}$ using angles separated by the optimal 22.5° and 67.5°.

$$|S_{HVT}| \le 2 \qquad |S_{QM}| \le 2\sqrt{2}$$

From this logic, a result bearing a value higher than two would prove the quantum mechanical interpretation to be legitimate whereas a value lower than two would be inconclusive to either interpretation.

Data and Results

To optimize the *S* value for conclusive results, our experiment used polarizer angles of to collect coincidence counts. After extensive experimentation and calculation, our final Bell Inequality came out to be

$$S = 2.729 \pm 0.024.$$

This result violates Bell's Inequality by more than thirty standard deviations, conclusively establishing the quantum mechanical interpretation as a legitimate description of polarity. Rather than having some predetermined polarity that we are unable to measure currently, photons have probabilities for certain polarities and only decide which polarity exactly when we consciously measure them. This probabilistic notion goes against many deterministic philosophies over thousands of years and could change the way we view the world. Subject to approval from the university, further research on this subject will be carried out in a thesis course next semester, for much more is left to be learned in this strange field of research.