Summer Research

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The Zeeman Effect in Mercury

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Purpose

The purpose of this experiment was to observe the effect of a magnetic field on the emission spectrum of Mercury, and determine whether or not the observations agreed with the predictions made by quantum mechanics.

The Apparatus

In this experiment we attempted to observe spectral lines whose wavelength difference was on the order of 10^{-12} m. The challenge was to build a spectrometer with enough resolving power to observe such small differences in wavelength. The spectrometer used in this experiment is outlined below.

Diagram of the Apparatus

Key:
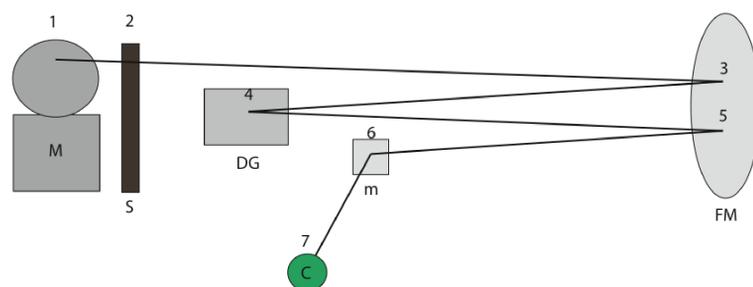
- M-Magnet
- S-Slit
- FM-Concave Focusing Mirror
- DG-Diffraction Grating
- m-Plane Mirror
- C-Camera

Path of Light through the Apparatus:

1. Light is generated in a gas discharge tube which is located between the poles of the magnet.
2. The light then passes through the slit.
3. After passing through the slit the light is reflected by the focusing mirror. The slit is located at the focal length of the focusing mirror, and as a result the light rays are collimated after reflecting from the focusing mirror.
4. The collimated rays then travel to the diffraction grating. Assuming that the diffraction grating is oriented correctly, light of the desired wavelength is diffracted back to the focusing mirror. The diffracted rays are still collimated.
5. The light then reflects off the focusing mirror. The incoming rays are collimated, and therefore the rays converge to form an image at a distance of the focal length away from the focusing mirror. This is precisely where the camera is located.
6. Light reflects off the plane mirror towards the camera.
7. The rays form an image at the camera lens.

Normal Zeeman Effect

In order to understand the Zeeman effect it is useful to use a semi-classical example of a single electron orbiting a nucleus in a circular path. The electron has magnetic dipole moment μ due to its orbit, which is:

1)
$$\mu = \frac{-e}{2m}L$$

where L, e, and m are the angular momentum, charge, and mass of the electron respectively. If a magnetic field B is applied at an angle θ with respect to the direction of the magnetic dipole, a torque τ of the following magnitude will be exerted to the dipole:

2)
$$\tau = \mu B \sin\theta$$

There is potential energy associated with how closely the magnetic dipole moment and the external magnetic field line up. Taking the potential energy U to be zero when $\theta = \pi/2$ and using equation 1) and 2), the potential energy of the magnetic dipole due to an external magnetic field is:

3)
$$U = \int_{\frac{\pi}{2}}^{\theta} \tau d\theta = \int_{\frac{\pi}{2}}^{\theta} \mu B \sin\theta d\theta = -\mu B \cos\theta = \frac{e}{2m}LB\cos\theta$$

Finally, if we take the direction of B to be the z-axis, then quantum mechanics predicts that the component of the angular momentum in the direction of the z-axis is quantized by the magnetic quantum number M_L. That is:

4)
$$L\cos\theta = M_L\hbar$$

Where M_L is an integer that ranges from $-L$ to L (L being the angular momentum quantum number). Therefore the potential energy, called the *Zeeman energy*, is quantized as well:

5)
$$U = M_L\left(\frac{e\hbar}{2m}\right)B$$

The Zeeman energies add to the standard energy levels of the atom, activating new energy levels and transitions that are not present without an external magnetic field. The result is a beautiful splitting pattern.

As a final note it is important to consider what are called the *selection rules* of the transitions. Quantum mechanics predicts that the magnetic quantum number M_L can only change by -1, 0, or 1 in a transition. If the change in M_L is -1 or 1 then then light given off will be polarized perpendicular to the magnetic field (σ polarization), and if the change in M_L is 0 then the light will be polarized parallel to the magnetic field (π polarization).

To summarize, an external magnetic field activates new energy levels in the atom which differ from the standard energy levels by the Zeeman energy given in Equation 5). These new energy levels, along with the selection rules that govern which transitions are allowed, result in splitting patterns of spectral lines. The yellow line of wavelength 5790.65 Å in the mercury spectrum exhibits the normal Zeeman effect. A theoretical diagram and our experimental findings for this transition are shown below.

Mercury Yellow Line from Transition $^1D_2 \rightarrow {}^1P_1$ (wavelength 5790.65 Å)

Theoretical Splitting Pattern | Observed Splitting Pattern
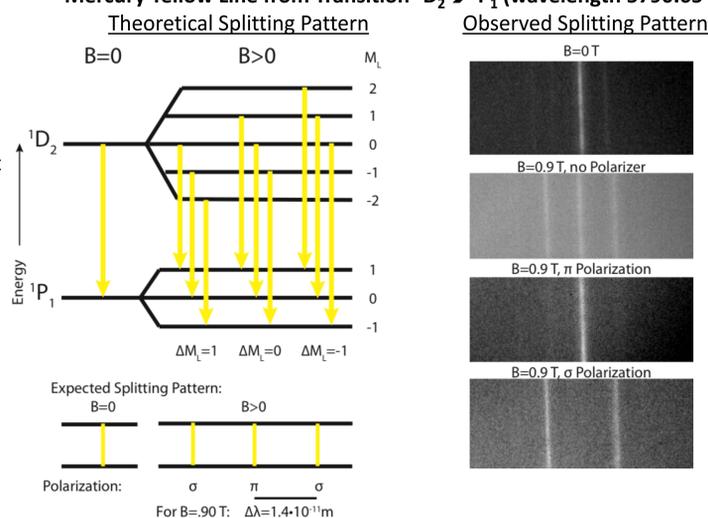
Anomalous Zeeman Effect

While the normal Zeeman effect triplet is observed in some transitions, more often than not complex splitting patterns, called anomalous Zeeman patterns, are observed. The normal Zeeman effect was discovered and explained in 1896 by Pieter Zeeman, but it took until 1925 for a theory that could explain the anomalous Zeeman effect. This theory stated that electrons have intrinsic angular momentum, called *spin*. The spin of the electron contributes to the magnetic dipole moment of the atom, and therefore to the Zeeman energy. When spin in included the Zeeman energy becomes

6)
$$U = M_J\left(\frac{e\hbar}{2m}\right)gB$$

where M_J is the total angular momentum magnetic quantum number, which includes spin, and g is the Landé *g-factor*:

7)
$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

where L, S, and J are the orbital angular momentum, spin angular momentum, and total angular momentum quantum numbers respectively.

The incorporation of the Landé g-factor allows for the Zeeman energy to take on different values than in the normal Zeeman effect, and as a result it agrees with the more complex splitting patterns of the anomalous Zeeman effect.
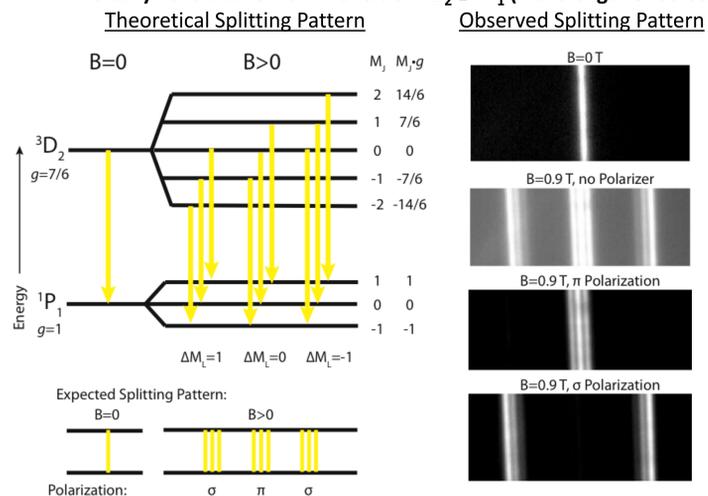
In fact, the normal Zeeman effect is a special case of the anomalous Zeeman effect in which the electron spin cancels out. When spin cancels out $S=0$, leaving $J=L$ and reducing g to unity. The Zeeman energy then reduces to the Zeeman energy predicted for the normal Zeeman effect in equation 5). This is precisely the case in the yellow line of wavelength 5790.65 Å in the mercury spectrum.

The theoretical and observed anomalous Zeeman splitting patterns are shown below for three lines of the mercury spectrum.

Mercury Blue Line from Transition $^3S_1 \rightarrow {}^3P_1$ (wavelength 4358.35 Å)

Theoretical Splitting Pattern | Observed Splitting Pattern
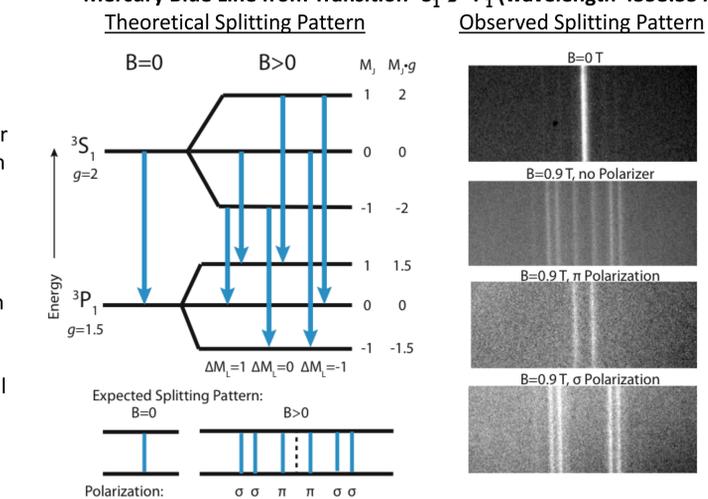
Mercury Yellow Line from Transition $^3D_2 \rightarrow {}^1P_1$ (wavelength 5769.59 Å)

Theoretical Splitting Pattern | Observed Splitting Pattern

Mercury Green Line from Transition $^3S_1 \rightarrow {}^3P_2$ (wavelength 5460.74Å)

Theoretical Splitting Pattern | Observed Splitting Pattern

References: Herzberg, Gerhard. *Atomic Spectra and Atomic Structure*. New York, NY: Dover Publications, 1944.