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Stochastic Optimal Harvesting Applied to Fisheries

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Stochastic Optimal Control Applied to Fisheries

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Overview:

We investigate alternative regularization schemes within a discrete time stochastic fishery model using stochastic dynamic programs.

Population Growth:

We suppose that harvesting occurs over a short period where natural death is negligible. For the n^{th} time period, we let

$$X_n = \text{Population biomass},$$
$$h_n = \text{Harvest size}.$$

We let *f(x)* denote the average stock recruitment function and Z_n be a random variable. Then the population size X_{n+1} is given by

$$X_{n+1} = Z_n f(X_n - h_n).$$

We use the logistic difference equation to model our average stock recruitment function

$$\Delta x = rx\left(1 - \frac{x}{K}\right),$$

for intrinsic growth rate *r* and carrying capacity *K*.

Cost Function:

We examined two cost functions, one proportional to Effort (E) and one quadratic in E:

$$C_1(E) = k_1 E,$$

$$C_2(E) = k_1 E + k_2 E^2.$$

We approximate these using $\Delta h \sim \dot{h} = qEx$, and express our unit cost functions as

$$c_1(x) = \frac{k_1}{qx},$$
$$c_2(x) = c_1(x) + \frac{k_2}{(qx)^2}.$$

Optimal Control:

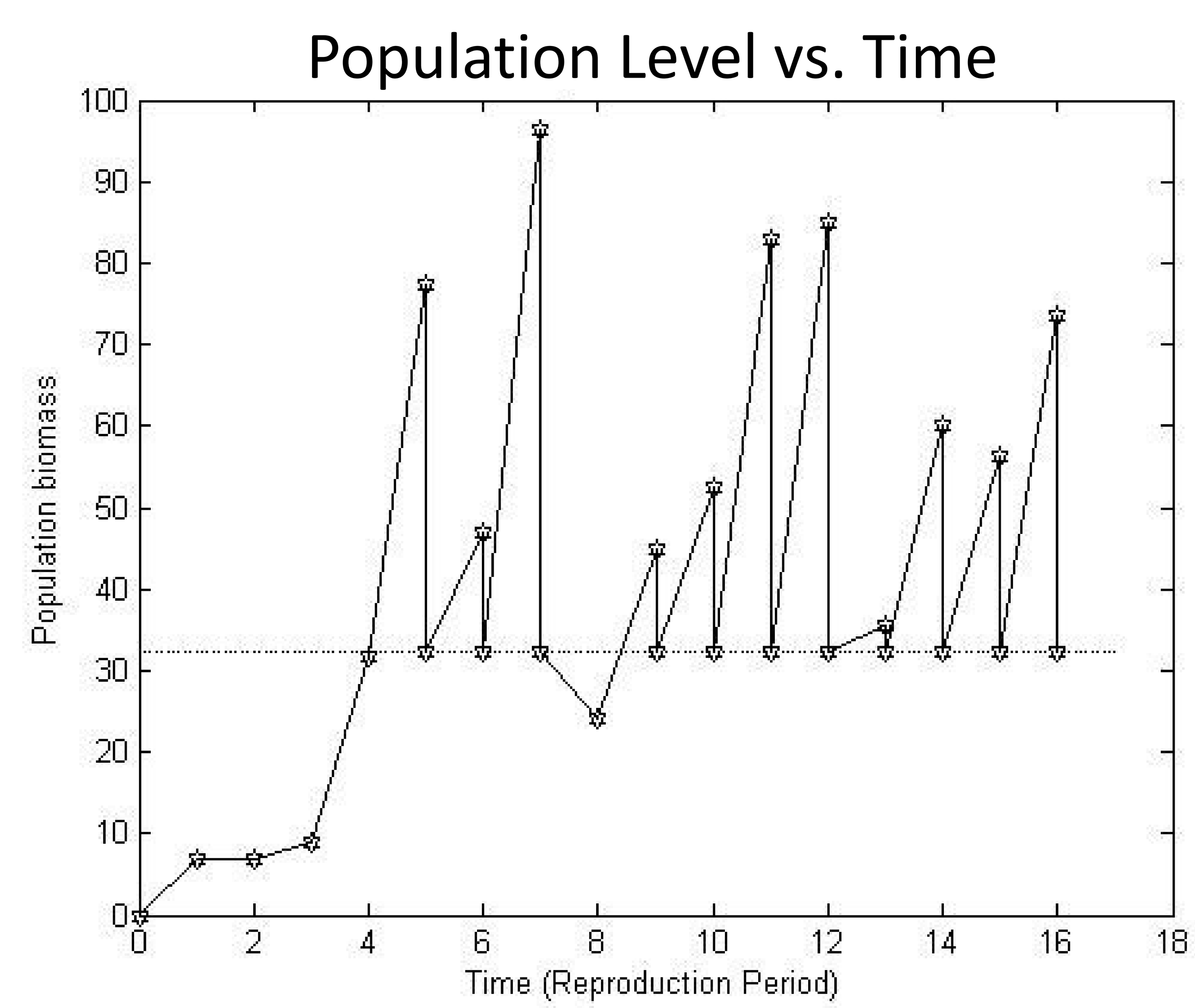
For an annual discount factor α, the optimal harvesting regime is one that maximizes expected total discounted revenue over an infinite time horizon:

$$E\left\{\sum_{i=1}^{\infty}\left\{\alpha^i\left[ph_n - \int_{X_n-h_n}^{X_n} c(t)dt\right]\right\}\right\}$$

From Reed[1], the optimal harvesting regime is of the form

$$h_n(X_n) = \begin{cases} 0, & X_n \le S \\ X_n - S, & X_n \ge S. \end{cases}$$

Population Control Under Optimal Escapement Regime:



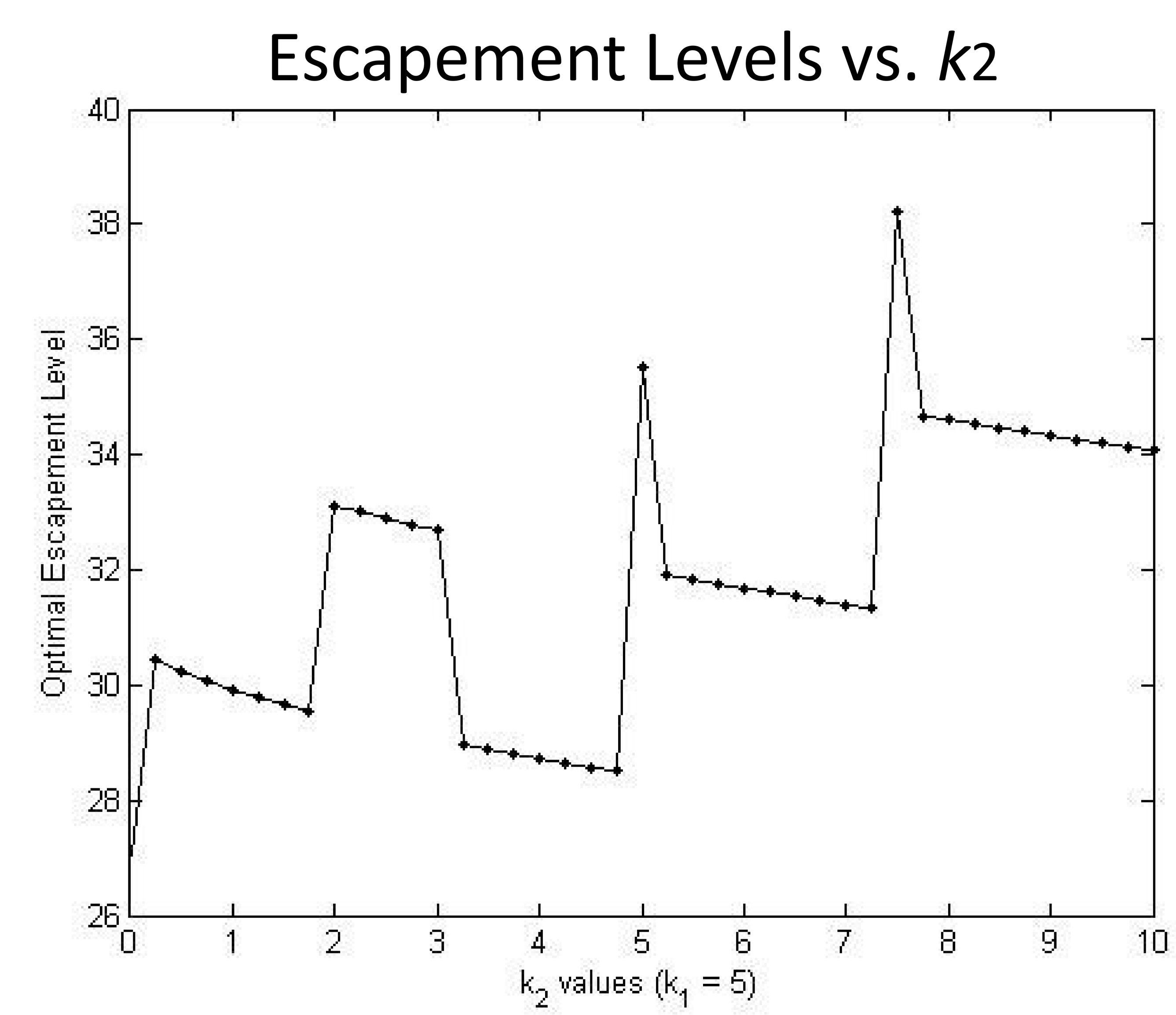
Stochastic Dynamic Program:

We constructed a SDP to calculate the Optimal Escapement Level, *S*. Our SDP used backwards recursion to identify *S* for various parameter values with both cost functions. Once *S* was found, we used forward recursion to plot simulation results for specific instantiations of the noise process.

Analysis and Future Work:

While an constant escapement regime is optimal under the given assumptions for any parameter values k_1 and k_2, the escapement level *S* does not appear to grow monotonically with the parameter values. The relationship between k_1 and *S* appears linear, but the relationship between k_2 and *S* appears more complicated, as displayed below. Future research would examine why this phenomenon may occur.

Impact of k_2 on Escapement Levels:



Works cited:

Reed, William J. "Optimal Escapement Levels in Stochastic and Deterministic Harvesting Models." *Journal of Environmental Economics and Management.*. 6. (1979): 350-363. Print.

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