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Vibrational Patterns in Curved Metal Plates

Samuel D. Berling
University of Puget Sound, sberling@pugetsound.edu

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Introduction

The musical saw is an instrument played by using one hand to bend the saw into an s-shape and using the other hand to bow the saw.

The musical saw inspired this more general study on the vibrational patterns of curved metal plates. I sought to identify factors affecting the vibrational patterns in curved metal plates.

Materials and Methods

I used an Electronic Speckle-Pattern Interferometry (ESPI) system [Figure 1] to image the plates under certain conditions, and I used COMSOL Mutliphysics software [Figure 2] to model the plates under those same conditions.

The ESPI combined with a spectrum analyzer was my method for finding resonant frequencies and normal modes. I focused on the modes with two horizontal nodal lines (called "(2, X)" modes) as these are generally the modes we hear when playing an instrument such as the musical saw.

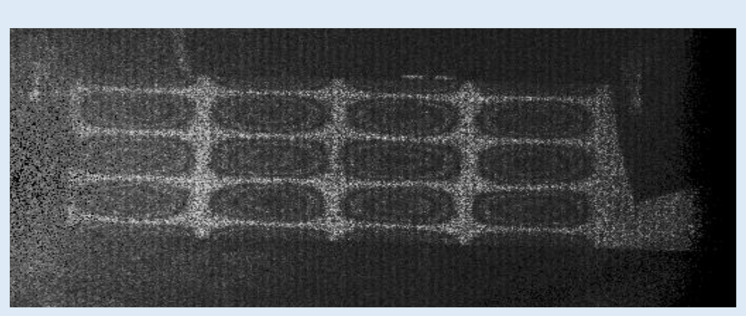

Figure 1 ESPI Image of a plate vibrating at its (4,3) Mode. Mode numbers are denoted by the horizontal (4) and vertical (3) nodal lines, excluding the ends. White lines in ESPI images represent the nodal lines.

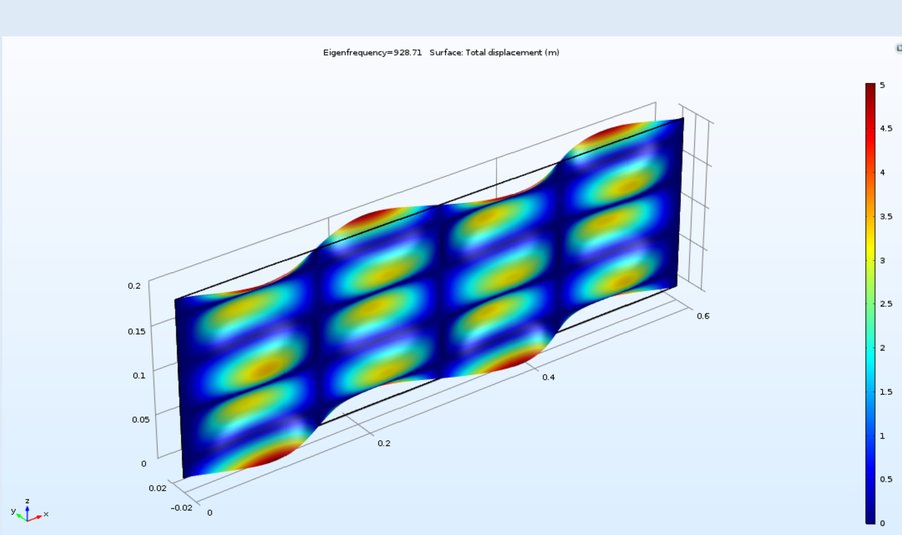

Figure 2 A COMSOL image representing the same (4,3) Mode as Figure 1. The blue lines in COMSOL images represent the nodal lines.

Data

The factors I investigated were (1) how plate size affected the frequency of a normal mode, (2) how taper affects resonant frequency, (3) how curvature affected the confinement of a vibrational pattern to a localized region on a plate, and (4) whether the stress of a blade being bent affects the resonant frequency. I was particularly interested in the stress factor because one of the few papers on this topic assumes stress played no role in its analysis [1].

Plate Size

To investigate how plate size affects the resonant frequencies of a given normal mode, I used three plates of two widths and two lengths [Figure 3].

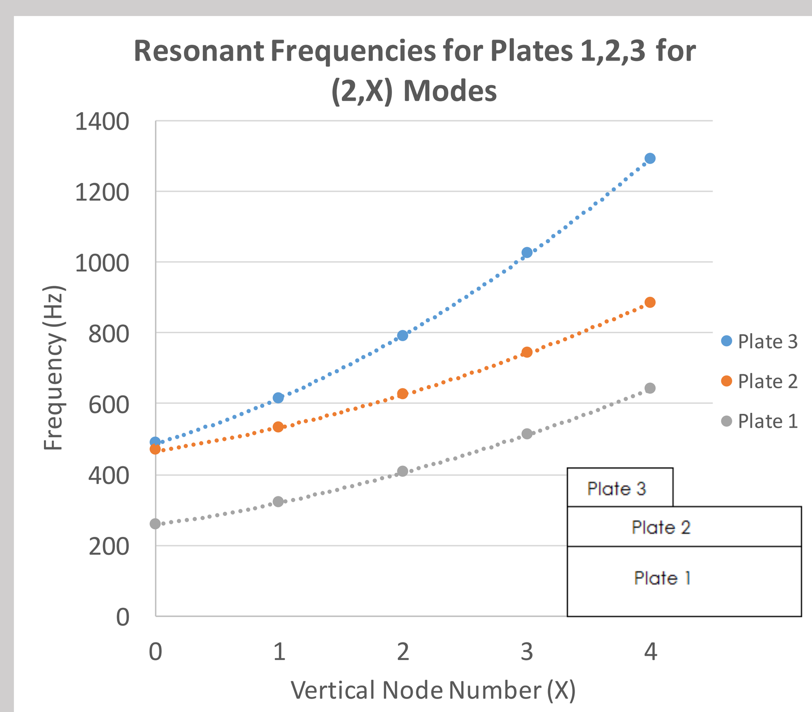

Figure 3 A plot of resonant frequencies for (2, X) Modes. Plate 3 is the smallest, Plate 1 is the biggest, and Plate 2 is in between.

Taper

I cut three plates to investigate taper, Plates 6, 7, and 8. Plates 6 and 8 were rectangular with Plate 6 being half the width of Plate 8. Plate 7 was tapered from the width of Plate 8 to the width of Plate 6 [Figure 4].

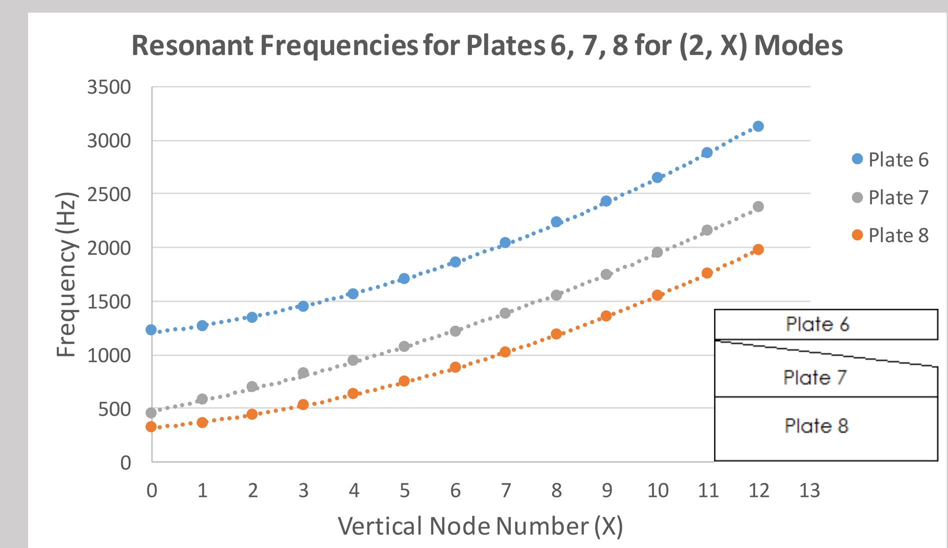

Figure 4 A plot of resonant frequencies of (2, X) Modes for a tapered plate and two rectangular plates.

Stress

To study the effect of stress, I compared the modes of a stressed curved plate (Plate 10) and an unstressed curved plate (Plate 9) [Figure 6].

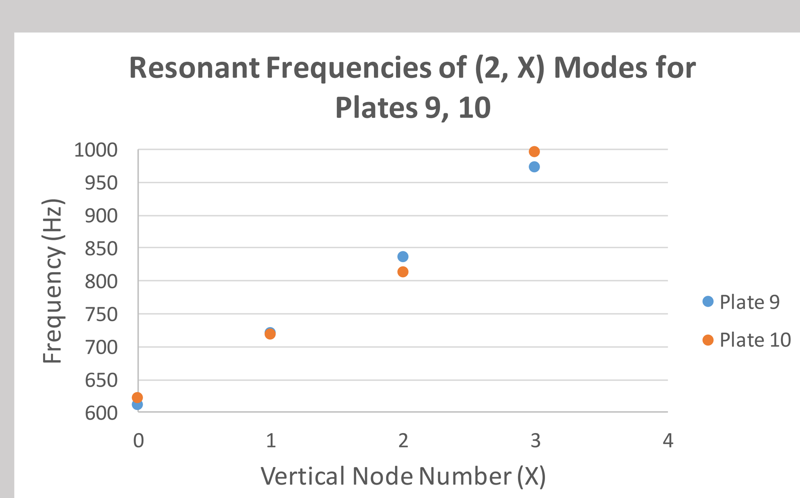

Figure 6 A comparison of resonant frequencies of (2, X) modes between a stressed plate (Plate 10) and an unstressed plate (Plate 9) of the same shape.

Curvature

To study the effect of curvature, I clamped a rectangular plate (Plate 11) to create an s-shape, similar to those used in the playing of a musical saw, I then traced the shape and used a plot digitizer to import it into Excel [Figure 5].

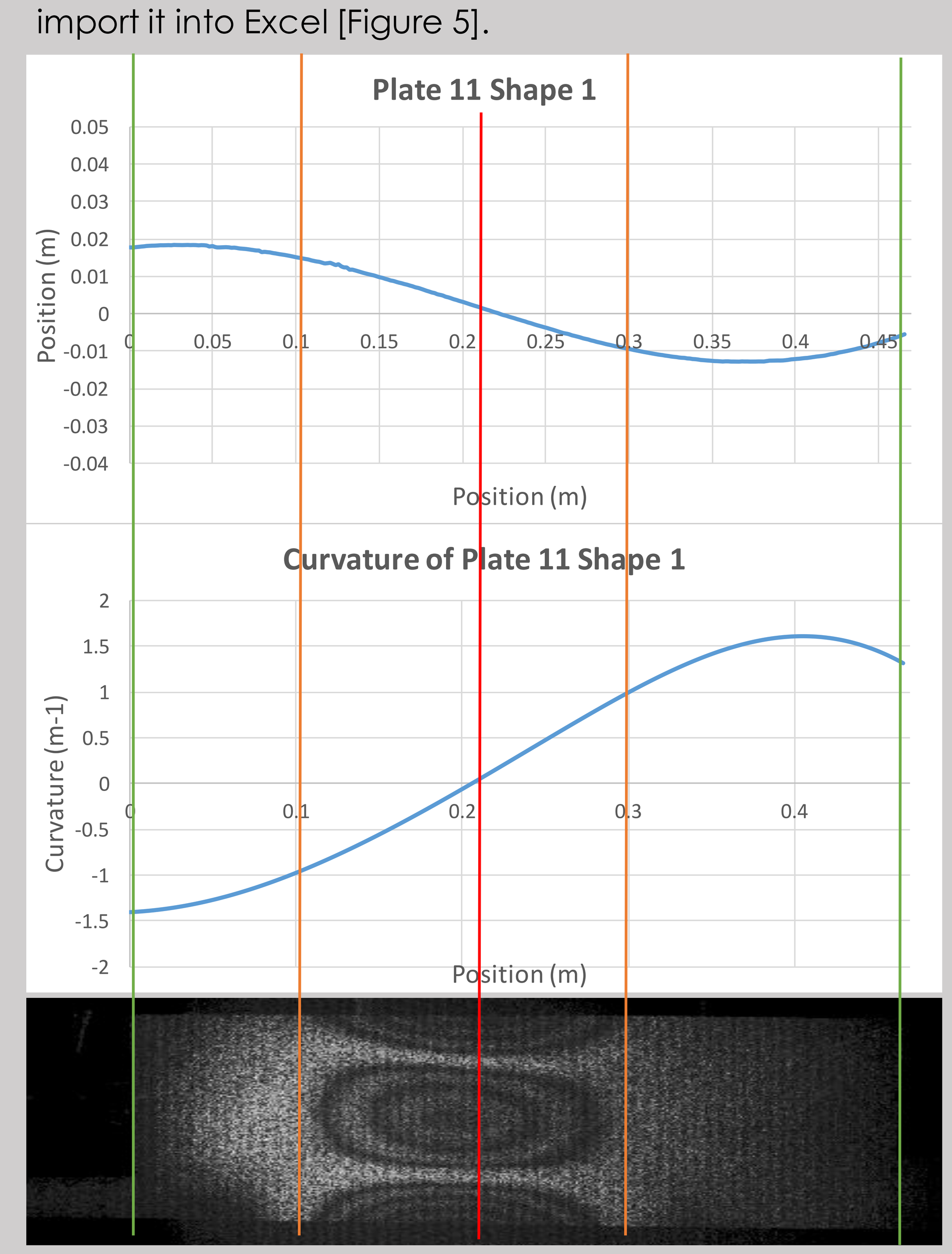

Figure 5 The top is a digitized tracing of the shape Plate 11's curve. The middle is a plot of the curvature of the shape. The bottom is an ESPI image of the plate resonating at the (2,0) mode. The green lines denote the edges of the plate. The orange lines denote the edges of the pattern of vibration. The red line denotes where the curvature is zero.

Qualitative Results and Analysis

- Figure 3 illustrates that a decrease in plate width significantly increases the resonant frequencies of a given horizontal mode set but as vertical nodal lines are added, the increase remains roughly the same.
- Figure 3 illustrates that a decrease in plate length produces a much smaller increase in the resonant frequency of the (2, 0) mode, but as vertical nodal lines are added, the difference in resonant frequency increases.
- Figure 4 shows that the resonant frequencies of a tapered plate (Plate 7) are in between those of the corresponding wide (Plate 8) and narrow (Plate 6) rectangular plates.
- Figure 6 shows that the effect stress has on the resonant frequencies is negligible for curved plates with none differing by more than 5%.
- The curvature study showed that a curved plate confines the vibrational patterns to a particular region on the plate. Figure 5 shows that the pattern occurs roughly where the curvature is less than 1 m^{-1} and that the center of the pattern occurs roughly at the inflection point of the the double curve.

References

[1] J. F. M. Scott; J. Woodhouse, "Vibration of an Elastic Strip with Varying Curvature" *Philosophical Transaction: Physical Sciences and Engineering*. Vol. 339, No. 1655 (1992): 587625.

Acknowledgments

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