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Matthew J. Moelter

Martin Jackson
University of Puget Sound, martinj@pugetsound.edu

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Formulas in Physics Have a "Standard" Form

Matthew J. Moelter, California Polytechnic State University, San Luis Obispo, CA
Martin Jackson, University of Puget Sound, Tacoma, WA

We discuss the importance of the ordering of symbols in physics formulas and identify implicit conventions that govern the "standard" form for how formulas are written and interpreted. An important part of writing and reading this form is understanding distinctions among constants, parameters, and variables. We delineate these conventions and encourage instructors to make them explicit for students.

In most cases the natural phenomena described by physical theories are represented in the language of mathematics. To succeed, students must understand this language both in terms of the underlying mathematics and how it is used in physics. Physics curricula typically require students to have mathematical (pre)co-requisites along with their physics courses. Mathematics, as used by physicists, is a language complete with a grammar and notational conventions. In particular, formulas used by physicists have particular conventions that help to expose different aspects of the physical content. Romer[1] has an excellent discussion of the process of "reading the equations" of physics. More recently, Hewitt[2] encourages the conceptual understanding of the equations as a guide to problem solving. However, this type of understanding is only possible if one can read the language of the formulas. We suggest that there are implicit grammatical rules concerning the writing of formulas in physics, and that these conventions should be explicitly identified and taught to enhance student understanding.

Example of a student's formula

A student's improper use of mathematical notation can readily lead to confusion for both the student and instructor. Many of us are familiar with the following difficulties: mixing upper and lower case (M versus m); failing to properly indicate the vector nature of a quantity ($\mathbf{F} = ma$); or careless use of sub/superscripts, $m2$ versus m^2 versus m_2. These are examples of incorrect symbol usage and students are typically instructed on their proper use. There is, however, a more subtle issue regarding the way symbols are arranged in a formula, even when all the symbols are present and correct.

Consider the following exam problem posed to students in an introductory calculus-based course:

(a) Use Gauss's law to find the electric field outside of an object with a spherically symmetric charge distribution and total charge 2Q.

(b) Sketch the magnitude of the electric field outside the sphere as a function of the distance from the center of the sphere, E_r versus r.

A student solved the problem correctly using the appropriate reasoning (concentric spherical Gaussian surface, constant electric field magnitude on surface, area of sphere, charge enclosed, etc.). Here is the student's answer for part (a),[3]

$$\left|\mathbf{E}\right| = \frac{2Q}{4\pi r^2 \varepsilon_0}\ . \qquad (1)$$

To the practiced eye this appears a bit odd. Why? The formula is not written in the "standard" form (see below). But does this really matter? For this student, yes.

Part (b) of the problem asked the student to make a sketch based on the result from the first part. The student's sketch was a horizontal line on properly labeled E_r-versus-r axes. During a face-to-face discussion after the exam, it became clear that the student did not "see" the factor of r^2 buried in the denominator when attempting to recognize and sketch the functional form of the spatial dependence of the field.

The student's answer, Eq. (1), contains all of the correct symbols in a mathematically acceptable position (products, numerator, denominator, etc.). From a mathematical standpoint the answer is exactly "right," by which we mean that if you substituted numbers for each of the symbols you would obtain the correct numerical result. However, colleagues with whom we shared examples such as this described the formulas as "awkward," or "difficult to interpret," or "confusing." What is it that makes them seem odd?

The "standard" form of physics formulas

Examining the displayed formulas in almost any physics text reveals a standard convention used when physicists write formulas. Let's look at two typical formulas:

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 \quad \text{and} \quad \mathbf{B} = \frac{\mu_0}{4\pi}\frac{Id\mathbf{l}\times\hat{\mathbf{r}}}{r^2}. \qquad (2)$$

The placement of each of the symbols in these formulas appears in an arrangement that follows canonical (but unwritten) rules. Almost all expressions used in physics texts conform to a "standard" order in which each of the terms is written in the form

$$\text{(constants)(parameters)(variables).} \qquad (3)$$

If the term is a fraction, the ordering can be applied to the numerator and denominator separately or factored into a product. The ordering is also applied within the arguments of transcendental functions.

A first step in helping students learn to read and write

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formulas using this standard order is to help them classify the various quantities at hand into the categories of *constants, parameters*, and *variables*.

Constants: These are mathematical or physical quantities that never change such as

- numbers in fractional or decimal form: $\frac{1}{2}$, 4, 2.43, …
- named numbers: e, π, γ, …
- physical constants:[4] G, ε_0, c, k_B, $\hbar$ …

If more than one "constant" appears, then they are usually ordered in the sequence given above, e.g., in $4\pi\varepsilon_0$, the order is number/named number/physical constant.

Parameters: These "quantities are constant for a particular experimental run but can change from run to run."[5] Parameters are quite important and many times crucial to our physical understanding. Let's take the example of a ball dropped from rest near the surface of a planet, where the position is given by

$$y = h_0 - \tfrac{1}{2}gt^2.$$

To appreciate the significance of the parameters h_0 and g, think in terms of making measurements. For each experimental run, the quantities h_0 and g have a fixed value, while t and y change during the experiment. In another experimental run, we might change the initial height, so h_0 would be different, or we could go to a different planet and g would change. Thus, h_0 and g are not "constants" in the same way that π or c represent particular mathematical or physical constants. So, a parameter is a quantity such as h_0 or g that is constant for a particular experimental run, but might change from run to run.

Variables: These are usually the quantities of most physical interest and correspond to time, position, electric field, or …. They are most like the inputs and outputs of the mathematical expression of functions.

Of course one person's parameter might be another person's variable. The distinction generally depends on the context and which quantity is of mathematical or physical interest.

Be explicit with students: The "standard" ordering matters

When physicists discuss phenomena using mathematics, the "standard" ordering can be a crucial part of the discussion. Typically we want to understand the essential behavior of one physical quantity as a function of some other quantity.

For example, how does the electric field depend on distance or on charge? Specifically, consider the magnitude of the electric field for a long, thin rod with uniform linear charge density λ_0. We show three versions of the formula that exhibit different emphases [as indicated in square brackets]:

$$|\mathbf{E}| = E = \frac{\lambda_0}{2\pi\varepsilon_0}\frac{1}{r} \qquad \text{[everything]} \qquad (4)$$

$$|\mathbf{E}| = E = (\ \ )\frac{\lambda_0}{r} \qquad \text{[parameter/variable]} \qquad (5)$$

$$|\mathbf{E}| = E = (\ \ )\frac{1}{r} \qquad \text{[variable]} \qquad (6)$$

From the second version the experienced reader can readily see that the field doubles upon doubling the charge density or halving the distance. This "standard" form is important when physicists interpret and discuss the physical meaning of a formula.

While some students will write formulas properly by imitation or through repetition, many will not. Because there is such a strong adherence to the formula conventions, we can help students communicate more effectively by being explicit about these conventions. To help students better use and appreciate the power of the mathematical description of the world, we suggest that instructors (and textbook authors) explicitly discuss the conventions regarding the "standard" form and the ordering of quantities in formulas. This might be done by following a treatment similar to that presented here. This should include a discussion surrounding the distinctions among constants, parameters, and variables.

As with most things, achieving facility requires repetition and practice with timely feedback. Therefore, instructors should provide explicit practice in working with the standard form and should enforce the standard form when evaluating student work. For example, instructors could ask students to rewrite Eq. (1) in "standard" form and then expect one of the following:

$$|\mathbf{E}| = \frac{2Q}{4\pi\varepsilon_0}\frac{1}{r^2} = \frac{1}{2\pi\varepsilon_0}\frac{Q}{r^2}. \qquad (7)$$

In evaluating student work, we might invent a new copyediting symbol such as "NSF" for "not in standard form."

Conclusion

Physics formulas follow implicit conventions and are written in a "standard" form. We have identified the elements of this form consisting of constants, parameters, and variables, in that order. To assist students, instructors are encouraged to make this convention explicit and encourage its use.

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2. Paul G. Hewitt, "Equations as guides to thinking and problem solving," *Phys. Teach.* **49**, 264 (May 2011).
3. Discussion and follow-up interview with student in calculus-based introductory course for science and engineering students.
4. In advanced courses, especially those in high energy or particle physics, common practice is to use a system of units such that $\hbar = 1$ or $c = 1$ and the like. However, this is not common at the introductory or intermediate level, so we ignore it here.
5. Regarding parameters we closely follow the discussion in Andrew F. Rex and Martin Jackson, *Integrated Physics and Calculus* (Addison-Wesley, New York, 2000), p. 39.

Martin Jackson *has been a professor of mathematics at the University of Puget Sound since 1990. In collaboration with colleagues from physics, he has team-taught a course that integrates the introductory physics and calculus sequences. The text* Integrated Physics and Calculus, *co-authored with Andrew Rex, is based on that course.*
Department of Mathematics and Computer Science, University of Puget Sound, Tacoma, WA 98416; martinj@pugetsound.edu

Matthew Moelter *is a professor of physics at Cal Poly and teaches all manner of introductory physics. He has particular interests in physics education and condensed matter.*
Department of Physics, California Polytechnic State University, San Luis Obispo, CA 93407; mmoelter@calpoly.edu