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By Christián Carlos Carman* and James Evans**

ABSTRACT

In the third century B.C.E., Eratosthenes of Cyrene made a famous measurement of the circumference of the Earth. This was not the first such measurement, but it is the earliest for which significant details are preserved. Cleomedes gives a short account of Eratosthenes' method, his numerical assumptions, and the final result of 250,000 stades. However, many ancient sources attribute to Eratosthenes a result of 252,000 stades. Historians have attempted to explain the second result by supposing that Eratosthenes later made better measurements and revised his estimate or that the original result was simply rounded to 252,000 to have a number conveniently divisible by 60 or by 360. These explanations are speculative and untestable. However, Eratosthenes' estimates of the distances of the Sun and Moon from the Earth are preserved in the doxographical literature. This essay shows that Eratosthenes' result of 252,000 stades for the Earth's circumference follows from a solar distance that is attributed to him. Thus it appears that Eratosthenes computed not only a lower limit for the size of the Earth, based on the assumption that the Sun is at infinity, but also an upper limit, based on the assumption that the Sun is at a finite distance. The essay discusses the consequences for our understanding of his program.

E RATOSTHENES IS THE EARLIEST GEOMETER whose method for calculating the circumference of the Earth has been preserved. Earlier writers had proposed various values for the circumference, but we have no way to know what their methods or arguments were. Aristotle, writing around 350 B.C.E., remarked that certain mathematicians had found a circumference of 400,000 stades. And Archimedes, writing about a century later, noted that in his day 300,000 stades was a well-known value.[1] Eratosthenes'

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[1] Aristotle, *De caelo* 2.14.298a19–20; Charles Mugler, ed., *Archimède: Oeuvres*, 4 vols. (Paris: Belles Lettres, 1970–1972), Vol. 2, p. 137; and, for an English translation of Archimedes, Thomas L. Heath, *The Works of Archimedes* (Cambridge: Cambridge Univ. Press, 1891), p. 222.

celebrity now is all the greater because the value he obtained (250,000 or 252,000 stades) happens to be reasonably close to the correct one.[2]

Regarding Eratosthenes' life, most that is plausibly known comes from the *Suda*, a Byzantine historical encyclopedia of about the tenth century. Eratosthenes was born in the 126th Olympiad (276–273 B.C.E.) at Cyrene, an old Greek settlement on the coast of Libya. After spending some time in Athens, he was called to Alexandria by King Ptolemy III (reigned 247–222 B.C.E.), and he lived into the reign of Ptolemy V (205–180 B.C.E.). According to the *Suda*, because Eratosthenes came in second in every branch of learning, he was nicknamed "Beta," though some more generously styled him a second Plato.[3] Others called him "Pentathlos" (after the five-event athletic competition), because he competed in so many different fields. He wrote works of philosophy, poems, histories, a work on the constellations (the *Catasterisms*, which survives in abridged form), an account of the sects of the philosophers, and a work entitled *On Freedom from Pain*, as well as dialogues and literary criticism. He died at the age of eighty, having given up food because he was going blind.[4] Besides the works mentioned by the *Suda*, Eratosthenes is known to have written on chronology, as well as on the eight-year lunisolar cycle, and, of course, to have produced an ambitious *Geography*. A good deal is known about Eratosthenes' geographical work from comments (often critical) in the first two books of the *Geography* of Strabo. A papyrus list of directors of the Alexandria Library gives Eratosthenes as following Apollonius of Rhodes and preceding Aristophanes of Byzantium.[5]

Although Eratosthenes' account of his measurement of the circumference of the Earth has not survived, some centuries later Cleomedes summarized it in a reasonably detailed report.[6] According to Cleomedes, Eratosthenes assumes that Syene, *S*, and Alexandria, *A*, are located on the same meridian. (Refer to Figure 1, which we have provided for convenience, though no figures are included in Cleomedes' short discussion.) Because Syene is on the summer tropic circle, at noon at the summer solstice a vertical gnomon will cast no shadow: the Sun is straight overhead. But in Alexandria, at the same moment, the gnomon does cast a shadow and the Sun is located 1/50 of a circle (angle γ) below the zenith. Finally, the distance between the two cities is 5,000 stades. If we assume that the Sun is so far from the Earth that the rays falling on the two cities may be considered parallel, the zenith distance γ of the Sun at Alexandria is equal to angle *AES*, which is the latitude difference between Alexandria and Syene, as measured at the center, *E*, of the

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[2] Just *how* close depends on the value of Eratosthenes' stade, a subject on which there is a large literature but that need not concern us. See D. W. Roller, *Eratosthenes' "Geography": Fragments Collected and Translated, with Commentary and Additional Material* (Princeton, N.J.: Princeton Univ. Press, 2010), pp. 271–273.

[3] The text of the *Suda* actually has βήματα ("steps" or "platforms"), which most commentators take to be a mistake for βῆτα (the second letter of the Greek alphabet).

[4] *Suda* On Line: Byzantine Lexicography, http://www.stoa.org/sol/. Other ancient sources give slightly different ages for Eratosthenes at his death. Some modern historians doubt the 126th Olympiad and back up his birth by one or two Olympiads. See Roller, *Eratosthenes' "Geography"* (cit. n. 2), p. 8 n 43.

[5] *P. Oxy.* 1241, col. 2, lines 1–8. See B. P. Grenfell and A. S. Hunt, *The Oxyrhynchus Papyri*, Pt. 10 (London: Egypt Exploration Society, 1914), pp. 99–112. Strabo, *Geography* 1.2.2, adds a few biographical details. For a brief account of Eratosthenes' scientific work see Alexander Jones, "Eratosthenes of Kurēnē (*ca* 240–194 BCE?)," in *The Encyclopedia of Ancient Natural Scientists*, ed. Paul T. Keyser and Georgia L. Irby-Massie (London: Routledge, 2009), pp. 297–300. For a broad study of his life and work see Klaus Geus, *Eratosthenes von Kyrene: Studien zur hellenistischen Kultur- und Wissenschaftsgeschichte* (Munich: Beck, 2002).

[6] Cleomedes was the author of an elementary textbook of astronomy, treated from a Stoic perspective. The title of his book is uncertain, as are his dates. For the Greek text of Cleomedes' discussion of Eratosthenes' method see Robert Todd, *Cleomedis Caelestia (Meteōra)* (Leipzig: Teubner, 1990), 1.7, pp. 33–38. For an English translation see Alan Bowen and Robert Todd, *Cleomedes' Lectures on Astronomy: A Translation of "The Heavens" with an Introduction and Commentary* (Los Angeles: Univ. California Press, 2004), pp. 78–85.
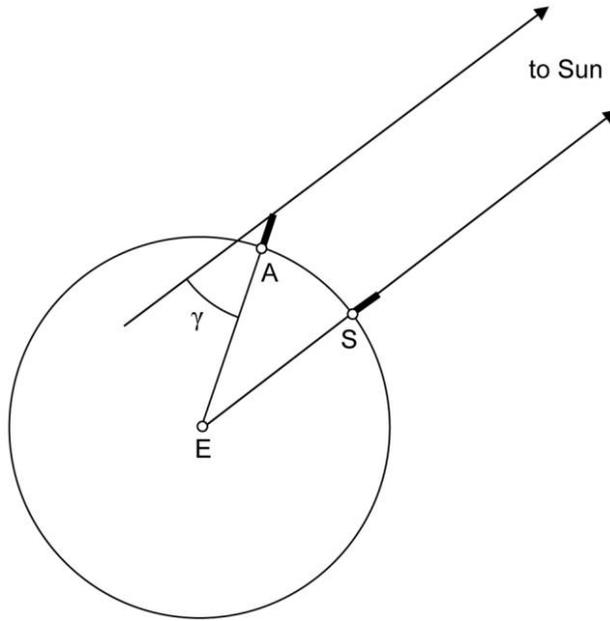
Figure 1. *The geometry of Eratosthenes' demonstration of the circumference of the Earth, as reported by Cleomedes.*

Earth. Then, since arc *AS* measures 5,000 stades and amounts to 1/50 of the circle, the entire circumference is equal to 50 × 5,000, or 250,000 stades.

Curiously, while Cleomedes and John Philoponus say that Eratosthenes' result was 250,000 stades, many other ancient sources give a value of 252,000 stades.[7] This is the case with Geminus, Hero of Alexandria, Strabo, Theon of Smyrna, Galen, Vitruvius, Pliny, and Martianus Capella, among others.[8] Not surprisingly, this difference of 2,000 stades has sparked the interest and imagination of historians. A common explanation is

[7] For Philoponus's report see Inna Kupreeva, trans., *Philoponus: On Aristotle Meteorology 1.1–3* (London: Bristol Classical, 2011), sec. 1.3.2, p. 44. Philoponus's source was the *Meteorology* of Arrianus.

[8] Hero of Alexandria (*Dioptra* 35), Theon of Smyrna (*Mathematical Knowledge Useful for Reading Plato* 3.3), Vitruvius (*On Architecture* 1.6.9), Pliny (*Natural History* 2.247), Galen (*Institutio logica* 12.2), Censorinus (*De die natali* 13.2), and Martianus Capella (*Marriage of Philology and Mercury* 6.596–598) all attribute a circumference of 252,000 stades to Eratosthenes. Strabo (*Geography* 2.5.7, 2.5.34) asserts that Eratosthenes was the source of the circumference of the Earth used by Hipparchus, which also was 252,000 stades. Geminus (*Introduction to the Phenomena* 16.6) gives 252,000 stades as the circumference of the Earth but does not attribute it to Eratosthenes; see James Evans and J. L. Berggren, *Geminos's "Introduction to the Phenomena": A Translation and Study of a Hellenistic Survey of Astronomy* (Princeton, N.J.: Princeton Univ. Press, 2004), p. 211. Adrian Gratwick translates many of the sources in "Alexandria, Syene, Meroe: Symmetry in Eratosthenes' Measurement of the World," in *The Passionate Intellect: Essays on the Transformation of Classical Traditions Presented to Professor I. G. Kidd*, ed. Lewis Ayres (New Brunswick, N.J.: Transaction, 1995), pp. 177–202. Translations are also available in Roller, *Eratosthenes' "Geography"* (cit. n. 2). Hipparchus is said by Pliny (*Natural History* 2.247) to have modified Eratosthenes' value, adding 26,000 stades to it; see also D. R. Dicks, *The Geographical Fragments of Hipparchus* (London: Athlone, 1960), frag. 38, pp. 88–89, and Dicks's discussion of the various opinions about this supposed modification on p. 153. This claim is not supported by any other ancient source. Irene Fischer, "Another Look at Eratosthenes' and Posidonius' Determinations of the Earth's Circumference," *Quarterly Journal of the Royal Astronomical Society*, 1975, *16*:152–167, proposes modifying the value to 16,000 stades.

that the value for the circumference was modified for the sake of convenience: 252,000 implies that 1° of latitude corresponds to an even 700 stades. Of course, the Greek adoption of the Babylonian division of the circle into 360 degrees is posterior to Eratosthenes, so if this is the correct explanation the modification must have been made later—and not by Eratosthenes. Or, since Eratosthenes is known to have divided the meridian into 60 parts in his geographical writing, and 252,000 is also evenly divisible by 60, perhaps a change for the sake of numerical convenience was made by Eratosthenes himself.[9]

Other authors, in order to explain the 252,000 stades, have proposed to modify the input values. One might imagine that, sometime after his first measurement, Eratosthenes obtained better measurements of the overland distance and/or the Sun's zenith distance and that these led to an improved value of 252,000 stades. Thus, Ludwig Öttinger proposed to modify the Sun's zenith distance at Alexandria to 25/1,260 of a circle instead of 1/50. Edward Gulbekian changed the distance between the two cities to 5,040 stades. And Oskar Viedebantt proposed to modify both: the zenith distance to 1/48 and the distance to 5,250 stades. Dennis Rawlins argued that Eratosthenes' calculation is based on a circular argument and also highlighted the esoteric character of the number 252,000: the same value is attributed by Pliny (*Natural History* 2.83) to Pythagoras for the distance from the Moon to the Sun. Adrian Gratwick points out that 5,040 (252,000 divided by 50) is the number proposed by Plato (*Laws* 737e–744d) as ideal for all the purposes of the state, because it is divisible by all the integers from 1 to 10. Gratwick also proposes an interesting reconstruction of Eratosthenes' calculation. Most proposals, however, try to resolve the inconsistency between the inputs and the output by modifying one or both of the input data. All such proposals have a certain arbitrariness about them and are therefore untestable.[10]

In what follows we will offer a simple and testable explanation of the 252,000-stade circumference. As we shall see, it is the circumference of the Earth that results from placing the Sun at a finite distance from the Earth, based on an Earth–Sun distance that is actually attributed to Eratosthenes in ancient sources. Thus it appears that Eratosthenes attempted to give an upper and a lower limit for the size of the Earth. He obtained 250,000 stades by placing the Sun at infinity and 252,000 by placing the Sun at a finite distance from the Earth. This will have much to teach us about the nature of Eratosthenes' program.

[9] The endorsement of the "convenient rounding" hypothesis by both J. L. E. Dreyer and Otto Neugebauer has contributed greatly to its popularity: J. L. E. Dreyer, *A History of Astronomy from Thales to Kepler* (New York: Dover, 1953), p. 175 (first published as *History of the Planetary Systems from Thales to Kepler* [Cambridge: Cambridge Univ. Press, 1906]); and Otto Neugebauer, *A History of Ancient Mathematical Astronomy*, 3 vols. (New York: Springer, 1975), pp. 734–735. See also C. M. Taisbak, "Posidonius Vindicated at All Costs? Modern Scholarship versus the Stoic Earth Measurer," *Centaurus*, 1974, *18*:253–269, esp. p. 261; and Fischer, "Another Look at Eratosthenes' and Posidonius' Determinations of the Earth's Circumference," p. 154.

[10] Ludwig Öttinger, *Die Vorstellungen der alten Griechen und Römer über die Erde als Himmelskörper* (Freiburg: Poppen, 1850), pp. 102–105; Edward Gulbekian, "The Origin and Value of the Stadion Unit Used by Eratosthenes in the Third Century B.C.," *Archive for History of Exact Sciences*, 1987, *37*:359–363; Oskar Viedebantt, "Eratosthenes, Hipparchos, Poseidonios," *Klio*, 1914, *14*:207–256; Dennis Rawlins, "Eratosthenes' Geodesy Unraveled: Was There a High-Accuracy Hellenistic Astronomy?" *Isis*, 1982, *73*:259–265 (on the supposed circularity of Eratosthenes' calculation); Rawlins, "The Eratosthenes-Strabo Nile Map: Is It the Earliest Surviving Instance of Spherical Cartography? Did It Supply the 5000 Stades Arc for Eratosthenes' Experiment?" *Arch. Hist. Exact Sci.*, 1982, *26*:211–219, esp. p. 216 (on the esoteric character of 252,000); and Gratwick, "Alexandria, Syene, Meroe" (cit. n. 8).

ERATOSTHENES' PROGRAM

We have no reason to think that the simple demonstration reported by Cleomedes was original with Eratosthenes, since Archimedes' source as well as those mathematicians mentioned by Aristotle must have done something similar. Consider, for example, the unattributed circumference of 300,000 stades mentioned by Archimedes. Cleomedes preserves what might be a portion of the original data for this calculation, though he, too, leaves the source unnamed.[11] In Lysimachia (near the Hellespont), the head of the constellation Draco is seen straight overhead. But in Syene (upper Egypt), Cancer is seen overhead. The distance between the cities is 20,000 stades. And the difference in declination between the two constellations is 1/15 of a circle (24°). Cleomedes does not draw the actual conclusion, which is that the circumference of the Earth must be $15 \times 20,000 = 300,000$ stades (though we do get this circumference reported in Archimedes). This is a crude estimate, with the whole constellation of Cancer treated as a point (or, perhaps, with "Cancer" standing in for the summer solstitial point[12]). The numerical estimates are poor—the actual difference in declination between the asterisms should be closer to 28° or 30° and the north–south distance between the cities something closer to 12,000 stades. In any case, the geometrical demonstration made by Cleomedes' source would necessarily have been very similar to Eratosthenes' demonstration. (See Figure 2.) The declination difference γ between Cancer and the head of Draco must be measured at some one place (let us say Lysimachia, L). The stars must be assumed to be very far away; so the lines of sight to Draco from L and Syene, S, at their meridian crossings are parallel to one another. Thus the diagram and demonstration would be virtually the same as that attributed to Eratosthenes (illustrated in Figure 1). The 300,000-stade circumference has sometimes been attributed to Dicaearchus of Messina (a student of Aristotle), but this is speculative and there is no way to know which pre-Archimedean astronomer or geographer was responsible.[13] For our purposes it does not matter. The key point is that there was nothing very original in the basic method attributed to Eratosthenes.

Moreover, when Cleomedes treats Eratosthenes' measurement, he actually presents two different calculations of the circumference of the Earth—one, by Eratosthenes, based as we have seen on observations of the Sun near the zenith, and a later one, by Posidonius, the Stoic philosopher, based on observations of the star Canopus, which is seen on the horizon at Rhodes, but ¼ of a sign above the horizon in Alexandria. Cleomedes explains Posidonius's method first, then remarks, "Such is the approach of Posidonius concerning the size of the Earth; but that of Eratosthenes involves a geometrical procedure (*geōmetrikēs ephodou*) and is considered to be somewhat more obscure."[14] Cleomedes then assures his readers that he can make Eratosthenes' account clear and goes on to give

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[11] Cleomedes 1.5.58–62; Todd, *Cleomedis Caelestia (Meteōra)* (cit. n. 6), pp. 28–29; and Bowen and Todd, *Cleomedes' Lectures on Astronomy* (cit. n. 6), p. 68. Cleomedes uses these measurements not to calculate the circumference of the Earth but to refute an alternative flat-Earth cosmology. Hence, as Bowen and Todd point out (*ibid.*, p. 68 n 16), the numbers in this passage should be used cautiously.

[12] This was the interpretation of Neugebauer, *History of Ancient Mathematical Astronomy* (cit. n. 9), p. 962 n 5.

[13] W. A. Heidel argued for caution in the absence of any decisive evidence for an attribution to Dicaearchus in *The Frame of the Ancient Greek Maps, with a Discussion of the Discovery of the Sphericity of the Earth* (New York: American Geographical Society, 1937), pp. 113–119. But the attribution has recently been renewed by Paul Keyser, "The Geographical Work of Dikaiarchos," in *Dicaearchus of Messana: Text, Translation, and Discussion*, ed. William W. Fortenbaugh and Eckart Schütrumph (New Brunswick, N.J.: Transaction, 2001), pp. 353–372.

[14] Cleomedes 1.7.48–50; and Todd, *Cleomedis Caelestia (Meteōra)* (cit. n. 6), p. 35.
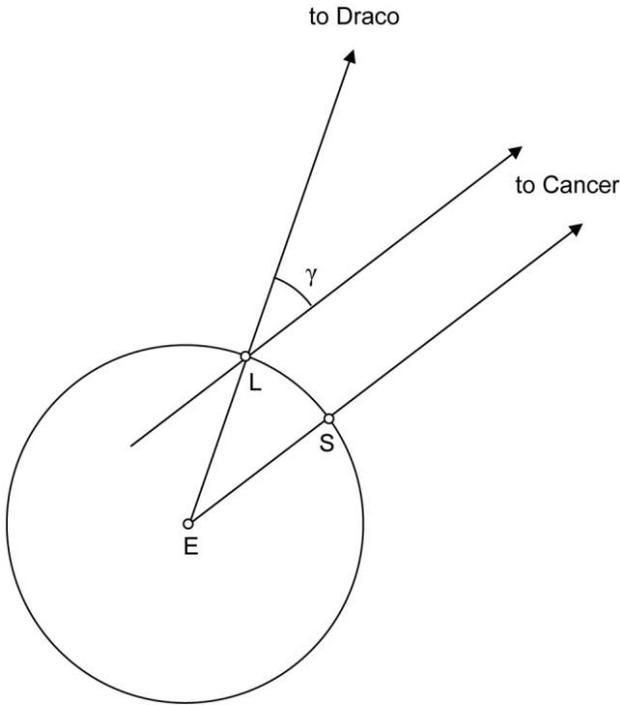
Figure 2. *Possible remains of an unattributed demonstration of the circumference of the Earth.*

the familiar explanation of Eratosthenes' calculation (summarized above). But the two geometrical situations postulated by Posidonius and Eratosthenes have exactly the same degree of complexity; indeed, Posidonius's situation is Eratosthenes' rotated by 90 degrees. Thus, Alan Bowen and Robert Todd suggest that in Cleomedes' description of Eratosthenes' procedure "*geōmetrikēs*" should be translated not as "geometrical" but, rather, as "geodesic," since Posidonius as well as Eratosthenes uses geometry.[15] On the other hand, Cleomedes' account of Posidonius's method is mostly just hand-waving. By contrast, his account of Eratosthenes' method involves a more detailed argument and invokes the theorem that a straight line intersecting parallel lines makes the alternate angles equal, and so forth. Perhaps this is all that Cleomedes meant when he said that Eratosthenes' procedure was geometrical and more obscure. But it is reasonably certain that Eratosthenes' calculation of the size of the Earth was part of a larger, more complex treatment, most of which has been ignored by Cleomedes as beyond the scope of his book—and probably beyond the mathematics he was comfortable dealing with. The mention that Eratosthenes' approach involves an obscure geometrical procedure then perhaps indicates that there were diagrams and nontrivial arguments drawn from them. The simple proofs of Posidonius and Eratosthenes that Cleomedes actually reproduces are expressed merely in words, with no diagrams.

Three ancient sources attest to the existence of a work by Eratosthenes that dealt with the size of the Earth and related matters and that was probably separate from his

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[15] Bowen and Todd, *Cleomedes' Lectures on Astronomy* (cit. n. 6), p. 78.

Geography.[16] Hero of Alexandria preserves a title: *Peri tēs anametrēsiōs tēs gēs* (On the Measurement of the Earth). Moreover, Hero informs us that in this matter "Eratosthenes worked rather more carefully than others."[17] This could possibly refer to a careful argument, based on travel times or other data, justifying the distance of 5,000 stades between the two cities, or to an account of how he measured the Sun's zenith distance at Alexandria—or to something else.[18] But it seems in any case to indicate something more detailed than the little geometrical demonstration that Cleomedes has given us.

Macrobius, in his *Commentary on the Dream of Scipio*, mentions a size of the Sun that he found *in libris dimensionum* (in the books about dimensions) of Eratosthenes, which is perhaps a reference to the same work.[19] According to Macrobius, Eratosthenes held that if we take the measure of the Earth and multiply it by 27 we will have the measure of the Sun. Scholars have disputed whether a linear or a volume measure is intended here. We shall return to this issue below. But Macrobius does at least provide evidence that some work of Eratosthenes dealt with the size of the Sun. Moreover, a finite figure for the size of the Sun clearly implies a finite value for the distance of the Sun.

Galen's short *Introduction to Logic* (*Eisagōgē dialektikē*, usually cited by the Latin title *Institutio logica*) uses some examples from astronomy to illustrate a discussion of categorical syllogisms. And here Galen remarks,

> For in inquiring whether Eratosthenes rightly showed the greatest circle on the Earth to have 252,000 stades, the question is of the size of the circle, or the magnitude, or the quantity, or however you wish to name it, just as whenever he seeks into how many stades is each of the tropics on the Earth, or, for each habitation, how large are the so-called arctic and antarctic circles, or by how many parts each habitation is distant from the north ["pole" probably to be understood].

These topics, then, we may plausibly take to have been treated in the same work of Eratosthenes that contained his figure for the circumference of the Earth: the circumference (measured in stades) of the terrestrial tropic circles, the latitudes of various key places on the Earth (equivalent to knowing the size of the arctic circle for a place), and the distances (in parts of the meridian) of various places from the terrestrial north pole.[20]

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[16] A detailed argument was made in support of this view by Amédée Thalamas, *La géographie d'Eratosthène* (Versailles: Barbier, 1921), pp. 65–78. See also Roller, *Eratosthenes' "Geography"* (cit. n. 2), p. 263.

[17] The title of Eratosthenes' work is given in Hero of Alexandria, *Dioptra* 35. See Hermann Schoene, ed., *Heronis Alexandrini Opera quae supersunt omnia*, Vol. 3: *Rationes dimetiendi et Commentatio dioptrica* (Leipzig: Teubner, 1903), pp. 302–303. Regarding the care Eratosthenes took see Nathan Sidoli, "Heron's *Dioptra* 35 and Analemma Methods: An Astronomical Determination of the Distance between Two Cities," *Centaurus*, 2005, *47*:236–258, on p. 240.

[18] Cleomedes (1.7.71–93) says that Eratosthenes used a sundial with a bowl (presumably a spherical dial) to judge the zenith distance of the Sun at Alexandria. This could make sense if he hung a plumb line at the end of the gnomon—it would offer a reliable way to measure the zenith distance. But Cleomedes goes on to say that there was also a sundial at Syene. This part is likely a fiction of Cleomedes' own invention. Martianus Capella (*Marriage of Philology and Mercury* 6.598) claims that Eratosthenes was provided by King Ptolemy's surveyors with the distance between Syene and Meroe. This is not confirmed by any other ancient source but, if true, could be another example of carefulness. See Jerker Blomqvist, "Alexandrian Science: The Case of Eratosthenes," in *Ethnicity in Hellenistic Egypt*, ed. Per Bilde (Aarhus: Aarhus Univ. Press, 1992), pp. 53–69.

[19] Macrobius, *Commentary on the Dream of Scipio* 1.20.9. See Mireille Armisen-Marchetti, ed. and trans., *Macrobe: Commentaire au Songe de Scipion*, 2 vols. (Paris: Belles Lettres, 2001–2003), Vol. 1, p. 114; and W. H. Stahl, trans., *Macrobius: Commentary on the Dream of Scipio* (New York: Columbia Univ. Press, 1952), p. 170.

[20] Carolus Kalbfleisch, ed., *Galeni Institutio logica* (Leipzig: Teubner, 1896), 12.2, pp. 26–27 (here and throughout this essay, translations are our own unless otherwise indicated). There is an English translation by

Galen then goes on immediately: "Also the size of the Sun and of the Moon and of their distances has been sought and demonstrated by the astronomers, just as also of eclipses, when they are not through the whole of the bodies, but a half or a third part or some other part. And the length of the days for each habitation has been sought and found, just as for the other [questions] mentioned." Now, in this addition it could be tempting to assume that Galen is still discussing the work of Eratosthenes, as was maintained by Heinrich Nissen.[21] This would entail adding a treatment of eclipses to Eratosthenes' work on the size of the Earth, along with the distances of the Sun and Moon and the variation of day length with latitude. But this would be too hasty, as Amédée Thalamas points out, since Galen is now speaking of "the astronomers" and these are standard astronomical topics.[22]

Fortunately, we have Macrobius's attestation that Eratosthenes did treat lunar eclipses in his book on dimensions.[23] Macrobius upbraids Eratosthenes for a circularity in his discussion: when Eratosthenes wants to demonstrate that the Sun is larger than the Earth, he draws on the evidence of the lunar eclipse, but when he wants to explain lunar eclipses he makes use of his demonstration of the size of the Sun. Macrobius is not competent in technical astronomy, but this is nevertheless a clear statement that Eratosthenes used lunar eclipses in his discussion of the size of the Sun. This is not surprising, since, after the work of Aristarchus of Samos, one would hardly attempt to deal with the sizes and distances of the luminaries without demonstrations based on lunar eclipses.

Finally, a number of ancient sources actually mention distances of the Sun and Moon attributed to Eratosthenes. Enough fragments survive of Eratosthenes' *On the Measurement of the Earth* to give us an idea of its scope and ambitions. Thalamas totaled up sixty-eight ancient citations that perhaps point to this work, of which nine concern astronomical matters such as the obliquity of the ecliptic and the distances of the Sun and Moon, forty-two concern the measurement of the Earth's circumference, and the remainder are divided among the zones, the winds, and the irregularities of the Earth's surface. Even when these are thinned out to avoid duplication or false attribution, as Thalamas argues, the scope of the work remains reasonably clear.[24]

In particular, a number of doxographical writers preserve values for Earth–Sun and Earth–Moon distances reported by Eratosthenes. According to Hermann Diels, much of this doxographical material descends from a compilation of tenets of philosophers that

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J. S. Kieffer, "Galen's 'Institutio Logica': English Translation, Introduction, and Commentary" (Ph.D. diss., Johns Hopkins Univ., 1964), p. 85. For an ancient Greek, the "arctic circle" is a circle on the sky with its center at the celestial pole and its radius chosen so that the circle just grazes the horizon in the north. Thus the arctic circle is the boundary between the stars that are circumpolar and those that rise and set. The angular radius of the arctic circle is equal to the terrestrial latitude of the place of observation.

[21] Kalbfleisch, ed., *Galeni Institutio logica*, p. 27; and H. Nissen, "Die Erdmessung des Eratosthenes," *Rheinisches Museum für Philologie*, 1903, *58*:231–245, esp. p. 232. Nissen was an early champion of the notion that Eratosthenes' measurement of the size of the Earth was contained in a work separate from the *Geography* and pointed to Hero's *Dioptra* 35 (recently published by Hermann Schoene [see note 17, above]) as evidence.

[22] Thalamas, *La géographie d'Eratosthène* (cit. n. 16), p. 126. Thalamas was reacting against Nissen's view that the contents of Eratosthenes' *On the Measurement of the Earth* could be read off in the extended passage of Galen. We note, however, that Armisen-Marchetti (*Macrobe: Commentaire au Songe de Scipion* [cit. n. 19], p. 193 n 440) again takes this passage of Galen as evidence that Eratosthenes treated the subjects mentioned— sizes and distances of Sun and Moon, eclipses, and so forth. Geus (*Eratosthenes von Kyrene* [cit. n. 5], p. 224) is cautious but seems inclined to reclaim this remark of pseudo-Galen as a reference to Eratosthenes.

[23] Macrobius, *Commentary on the Dream of Scipio* 1.20.9–10.

[24] Thalamas, *La géographie d'Eratosthène* (cit. n. 16), pp. 76–77. Roller (*Eratosthenes' "Geography"* [cit. n. 2], pp. 263–267) is more circumspect, printing nine fragments that have to do with the measurement of the Earth's circumference, the zones, and the obliquity of the ecliptic (and excluding the winds and the distances of the Sun and Moon).

was put together by a certain Aëtius around the beginning of our era.[25] Aëtius's compilation is not extant, but two different abridgements or reworkings of it survive and are important sources for early Greek philosophy of nature. One is the *Eclogae* (Extracts) of Joannes Stobaeus (probably fifth century C.E.). And one is ascribed to Plutarch (late first–early second century C.E.) and is commonly known as *De placitis philosophorum* (Opinions of the Philosophers). Diels refers to the latter as Plutarch's *Epitome* of the *De placitis* of Aëtius. (The attribution to Plutarch, the characterization of the work as an "epitome," and the name of Aëtius as the author of a compendium of physical opinions of the philosophers are all found in ancient sources.) The attribution to Plutarch has been widely doubted since the seventeenth century, and so the author of this abridgement is often styled pseudo-Plutarch. In any case, the pseudo-Plutarchian *Epitome* must have existed by the late second century C.E. Quotations or extracts from the *Epitome* were in turn made by (among others) Eusebius, the Bishop of Caesarea in Palestine (late third–early fourth century C.E.), in his *Praeparatio evangelica* (Preparation for the Gospel) and by Joannes Lydus (sixth century) in his *De mensibus* (On Months). The passages from all of these that bear on Eratosthenes' distances of the Sun and Moon are conveniently available in Diels's *Doxographi Graeci*.[26]

The sources in this tradition for the most part agree that Eratosthenes gave an Earth–Moon distance $d_{\mathbb{C}} = 780,000$ stades. If we use Eratosthenes' terrestrial circumference of 252,000 stades, we have for the radius, R, of the Earth 40,107 stades. Thus, the distance of the Moon is about 19.45 Earth radii. Although this is low by modern standards (the actual value is $d_{\mathbb{C}} \cong 60R$), it is fairly consistent with the premises of Aristarchus of Samos, who wrote in the first half of the third century B.C.E. Aristarchus does not give an explicit result for the distance of the Moon, but his premises lead to $d_{\mathbb{C}} = 20.1R$.[27] So Eratosthenes' lunar distance would not have seemed outrageous in the second half of the same century.

For the Earth–Sun distance $d_{\odot}$ there is a choice of texts. Stobaeus's version (*Eclogae* 1.26) reads "400 myriads of stades and 8 myriad stades"—that is, 4,080,000 stades.[28] (A

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[25] Hermann Diels, *Doxographi Graeci* (Berlin: G. Reimer, 1878). For a useful summary of Diels's case, and of objections to parts of it, see Jaap Mansfeld, "Doxography of Ancient Philosophy," in *The Stanford Encyclopedia of Philosophy*, Winter 2013 ed., ed. Edward N. Zalta, http://plato.stanford.edu/archives/win2013/entries/doxography-ancient/.

[26] Aëtius 2.31, in Diels, *Doxographi Graeci*, pp. 362–363. But see also the following newer editions: Guy Lachenaud, *Plutarque: Oeuvres morales*, Vol. 12, Pt. 2: *Opinions des philosophes* (Paris: Belles Lettres, 1993), p. 124; Jürgen Mau, *Plutarchi Moralia, Vol. V, Fasc. 2, Pars 1* (Leipzig: Teubner, 1971), pp. 97–98; and Karl Mras, *Eusebius Werke*, Vol. 8 in 2 parts: *Die Praeparatio Evangelica* (Berlin: Akademie-Verlag, 1956), Pt. 2, p. 417.

[27] Moreover, one can easily infer an upper and a lower limit for the lunar distance in Earth radii using Aristarchus's own propositions. In proposition 11 Aristarchus says, "The diameter of the Moon is less than two 45ths, but greater than one 30th, of the distance of the center of the Moon from our eye": Thomas L. Heath, *Aristarchus of Samos: The Ancient Copernicus* (Oxford: Oxford Univ. Press, 1913), p. 387. Therefore, $45/2\ D_{\mathbb{C}} < d_{\mathbb{C}} < 30D_{\mathbb{C}}$, where we use $D_{\mathbb{C}}$ for the diameter of the Moon. Also, in proposition 17 (*ibid.*, p. 409) Aristarchus says that the diameter of the Earth is bigger than 108/43 but smaller than 60/19 diameters of the Moon. Therefore, combining both results, the Moon's distance must be between 14.25 and 23.89 Earth radii. Eratosthenes' own result lies well within these limits.

[28] For Stobaeus's text see Diels, *Doxographi Graeci* (cit. n. 25), p. 363, right column, lines 1–3. Diels emends Stobaeus's text to correct the mistaken duplication of a word: σταδίων μυριάδας [μυριάδων] τετρακοσίας καὶ στάδια ὀκτάκις μύρια. With μυριάδων not suppressed (and thus 400 myriads of myriads of stades plus 8 myriad stades), the result would be impossibly large—over 4×10^{10}. Thus Diels's correction is not only reasonable but essential. The same text is printed in Wachsmuth's later edition of Stobaeus: Curtius Wachsmuth and Otto Hense, eds., *Ioannis Stobaei Anthologium*, 5 vols. (Berlin: Weidmann, 1884–1912), Vol. 1: *Anthologii libri duo priores qui inscribi solent Eclogae Physicae et Ethicae*, ed. Wachsmuth (1884; rpt., 1958), p. 223.

"myriad" is 10,000, the largest number expressible by a single word in Greek.) This reading finds some support in an apparent paraphrase of Aëtius found in Theodoret—"They reckon four hundred, and even more, myriads of stades from the Earth up to the Moon and from there up to the Sun"—although Theodoret does not mention Eratosthenes here.[29] One might worry that Theodoret (fifth century C.E.) is not the best source, as his goal in the surrounding passage is to attack the uselessness of pagan scientific dabbling. Still, in his testimony about the Presocratics, which we can compare with other sources, he is a reasonably faithful witness. Moreover, Theodoret makes considerable use of Aëtius (or a source descending from him) in the portion of his book dealing with ancient science.

In pseudo-Plutarch's *Epitome* of Aëtius the text has a gap, and so the Sun's distance is missing. Diels fills this by using the text of Eusebius (*Praeparatio evangelica* 15.53), supplemented by corresponding passages from Lydus (*De mensibus* 3.12) and from Pseudo-Galen (*Historia philosopha* 72). Diels's reconstructed text for pseudo-Plutarch reads: "(8 myriads and 400) myriads of stades"—that is, $80,400 \times 10,000 = 804,000,000$.[30] The version of Lydus is contorted and possibly corrupt, though it could be taken as supporting 804,000,000.[31] So it seems that a copying error or misunderstanding of the text affected one of the two textual traditions.[32] It is easy to see how this could have happened. In Greek it is often possible to omit a word that would otherwise repeat a word that appeared in a previous clause or phrase. But then a reader, coming across a text in which something like this might possibly have occurred, would have to infer whether to supply the missing word or not. Here it is "myriad" that caused the trouble, though there are different forms for the noun and the adjective. In any case, we are left with 4,080,000 stades and 804,000,000 stades as two possibilities for Eratosthenes' distance of the Sun. The 4 in either reading represents 400 myriads. The issue is whether the 8 represents 8 myriads or 8 myriads of myriads.

Various writers have favored either the larger or the smaller of the two possible Sun distances. T. L. Heath wrote, "The versions of Stobaeus and Joannes Lydus admit of 408 myriads of stades as an alternative interpretation [i.e., alternative to 804,000,000], but this figure obviously cannot be right."[33] Presumably Heath discarded this interpretation because the value would be rather small for an Earth–Sun distance. The ratio of the Sun's

[29] For Theodoret's text see Diels, *Doxographi Graeci*, p. 362, right column, footnote: καὶ τετρακοσίας ἀριθμοῦσι καὶ μέντοι καὶ πλείους σταδίων μυριάδας τὰς μὲν ἀπὸ γῆς μέχρι σελήνης, τὰς δὲ ἐκεῖθεν μέχρις ἡλίου. The text is the same in Joannes Raeder, *Theodoreti Graecarum affectionum curatio* (Leipzig: Teubner, 1904), p. 106. An English translation is available in Thomas Halton, *Theodoret of Cyrus: A Cure for Pagan Maladies* (New York: Newman, 2013), p. 99.

[30] Diels's reconstructed text for pseudo-Plutarch (*Doxographi Graeci*, p. 363, left column, lines 1–2) reads: σταδίων μυριάδας τετρακοσίας καὶ ὀκτακισμυρίας. The same text is printed by E. H. Gifford, *Eusebii Pamphili Evangelicae praeparationis libri xv*, 4 vols. in 5 parts (Oxford: Typographeo Academico, 1903), Vol. 2, p. 487, though some of the manuscripts differ. (However, we note that Gifford [*ibid.*, Vol. 3, Pt. 1, p. 912] translates this as "four millions and eighty thousand stades.") Mras gives the same Greek text in his recent edition of Eusebius's *Praeparatio evangelica* (cit. n. 26), Pt. 2, p. 417.

[31] Diels's text for Lydus (*Doxographi Graeci*, p. 362, left column, footnote) reads: ἀφεστάναι δὲ λόγος ἀπὸ τῆς γῆς κατὰ τὸν Ἐρατοσθένην τὴν μὲν σελήνην σταδίων μυριάδας ἑβδομήκοντα ὀκτώ, τὸν δὲ ἥλιον τετρακοσίας καὶ ὀκτάκις μυρίας. The text is the same in Ricardus Wünsch, ed., *Ioannis Laurentii Lydi Liber de mensibus* (Leipzig: Teubner, 1898), 3.12, p. 54.

[32] The passage from Pseudo-Galen, *Historia philosopha* 72, which poses its own difficulties, is quoted by Diels, *Doxographi Graeci*, p. 628. See the discussion of this text by Jaap Mansfeld, "Cosmic Distances: Aëtius 2.31 Diels and Some Related Texts," *Phronesis*, 2000, *45*:175–204, esp. pp. 187–188.

[33] Heath, *Aristarchus of Samos* (cit. n. 27), p. 340. However, we do not agree that the text of Lydus really admits of 4,080,000. And we note that Lachenaud, *Plutarque: Opinions des philosophes* (cit. n. 26), p. 124, translates Lydus as giving 80,400 myriads.

distance to the Moon's distance would be $d_\odot/d_\mathbb{C} = 4{,}080{,}000/780{,}000 = 5.23$, which is, indeed, somewhat small compared with other ratios known from antiquity—but not terribly so. Aristarchus put $d_\odot/d_\mathbb{C}$ between 18 and 20. According to Archimedes in the *Sand Reckoner*, Archimedes' father, Pheidias, favored $d_\odot/d_\mathbb{C} = 12$. (This is actually his value for the ratio between the solar and lunar diameters; but since the two bodies have about the same angular diameter, this would also be the ratio of the distances.) In addition—also according to Archimedes—Eudoxus had favored $d_\odot/d_\mathbb{C} = 9$. (Again, this value is actually stated as a ratio between diameters.) By contrast, the larger value for the Sun's distance is *very* far removed from the range of other ancient estimates: one would have $d_\odot/d_\mathbb{C} = 804{,}000{,}000/780{,}000 = 1{,}031$. *A priori*, it would be more plausible to assume the reading discarded by Heath than the one he preferred. Most recent scholars have inclined toward 4,080,000. Otto Neugebauer gives 4,080,000 stades for Eratosthenes' distance of the Sun but remarks that the textual tradition is corrupt. Jordi Pàmias i Massana and Arnaud Zucker give the same value without discussion in their recent edition of Eratosthenes' *Catasterisms*. Finally, Jaap Mansfeld makes a detailed examination of the whole passage (Aëtius 2.31) and its echoes in other Greek texts and in a scholium to the *Almagest*, as well as in the Arabic tradition, and concludes that, for Eratosthenes' distance of the Sun, "the original reading probably but far from certainly was 4,080,000. . . . Some of our sources read a myriad too much, others omit one. Dancing to the tune of the myriads easily leads to a *faux pas* one way or the other."[34]

The distance of the Sun expressed in Earth radii would be either $d_\odot = 101.7R$ (if we adopt the smaller of the two possibilities for the Sun's distance) or $d_\odot = 20{,}046R$ (if we adopt the larger). The former is a bit smaller than comparable ancient estimates. But the latter is much larger than other ancient values (though it happens to be rather close to the accepted modern value).

THE SIZE OF THE EARTH ASSOCIATED WITH ERATOSTHENES' SOLAR DISTANCE

Fortunately, all ambiguity can be removed, for only the solar distance of 4,080,000 stades is consistent with Eratosthenes' second value for the size of the Earth. Refer to Figure 3, which is similar to Figure 1 except that now the Sun is located at U. The obvious impact of a finite Earth–Sun distance is that we can no longer consider the Sun rays falling on the two cities as parallel. The Sun ray drawn from U to the tip of the gnomon at Alexandria is not parallel to the Sun ray drawn from U to the tip of the gnomon at Syene. Let α be the angle between the rays. Angle γ is still the Sun's zenith distance as observed at Alexandria (and equal to 1/50 of a circle, or 7.2°), but now this angle is no longer the same as the latitude difference β between Syene and Alexandria.

Now, δ and γ are supplementary angles, so

$$\delta = 180° - \gamma = 172.8°. \tag{1}$$

[34] Neugebauer, *History of Ancient Mathematical Astronomy* (cit. n. 9), p. 660; Jordi Pàmias i Massana and Arnaud Zucker, *Érathosthène de Cyrène: Catastérismes* (Paris: Belles Lettres, 2013), p. xv; and Mansfeld, "Cosmic Distances" (cit. n. 32), p. 188. However, Mansfeld proposes as a possible version of the original text: σταδίων μυριάδας τετρακοσίας καὶ ὀκτακισμυρίας. While we agree that 4,080,000 was probably the original reading, we think that Mansfeld's suggested Greek text actually better supports 804,000,000. It should be noted that there are several typographical errors on pp. 186–187 of Mansfeld's article, where 4,080,000 is printed as 40,080,000.
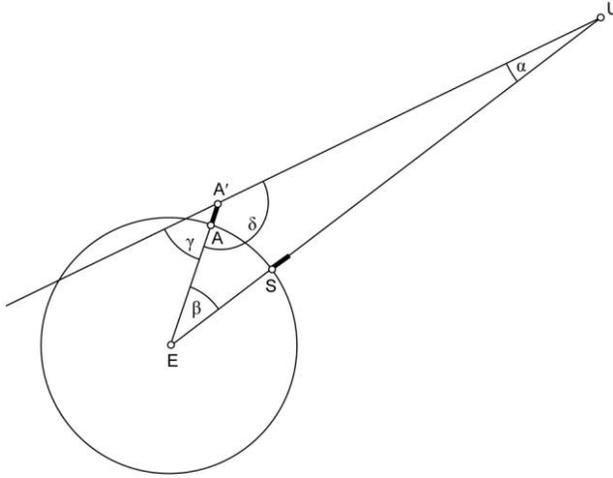
Figure 3. *Consequences of a finite solar distance for Eratosthenes' demonstration of the circumference of the Earth.*

Applying the law of sines in triangle $EA'U$, we have

$$\sin \alpha = \frac{EA}{EU} \sin \delta. \tag{2}$$

(Note that in Figures 2 and 3 we have drawn gnomons of finite height for clarity, but in doing the geometry one should let the heights of the gnomons be infinitely small, so A and A' coincide.) As we have seen, Eratosthenes had $EU/EA = d_\odot/R = 101.7$, so, using this together with (1) in (2), we find

$$\alpha = 0.07061°. \tag{3}$$

Then, since α, β, and $(180° - \gamma)$ must total $180°$, we have

$$\beta = \gamma - \alpha = 7.12939°. \tag{4}$$

And, finally,

$$\text{circumference of Earth} = 5{,}000 \cdot \frac{360}{7.12939} = 252{,}476 \text{ stades,} \tag{5}$$

which is within round-off error of Eratosthenes' published result of 252,000 stades. Of course, when we take into account the difficulties of third-century numerical geometrical calculation, we cannot be precisely sure what number Eratosthenes would have obtained—it could have been a bit more or a bit less than 252,476. But in ancient Greek mathematics, rounding off was done most often by simple truncation, as opposed to the modern style in which one checks to see whether the extra, unwanted digit is greater or less than 5. Thus 252,000 is quite a robust result from this calculation.

UPPER AND LOWER LIMIT CALCULATIONS

We suggest that both calculations were performed by Eratosthenes. The simple, well-known calculation, assuming that the Sun is at an infinite distance and giving an Earth circumference of 250,000 stades, must be understood as the lower limit: the Earth could not be smaller than that. The new calculation, assuming that the Sun is at a distance of around 102 Earth radii and giving a value of 252,000 stades, must be understood as the upper limit: the Earth could not be bigger than that. From the period just before Eratosthenes' work, we have Archimedes' bracketing of π between 3 10/71 and 3 1/7, as well as Aristarchus's result that the distance of the Sun is between 18 and 20 times the distance of the Moon. Thus, calculating an upper and a lower limit was within the normal range of mathematical procedures for Eratosthenes' day.

From the period after Eratosthenes, we have also the example of Hipparchus's work on the Moon's distance. According to a plausible reconstruction of his calculations, because Hipparchus did not know the Earth–Sun distance, he first assumed that the Sun is at an infinite distance and obtained one value for the Moon's distance, and then he assumed that the Sun is as close as it can be without producing a perceivable parallax and again calculated the distance of the Moon.[35] In this way, he obtained upper and lower limits for the lunar distance. We propose that Eratosthenes did something similar for the Earth's circumference. According to our proposal, Cleomedes explained just the simpler method—that is, the method for obtaining the lower limit. He omitted the upper-limit method that would have given the value of 252,000 stades; this would have been the "obscure" geometrical procedure that he mentions.

UPPER-LIMIT CALCULATION WITHOUT TRIGONOMETRY

Of course, Eratosthenes could not apply the law of sines, because trigonometry was not yet available, but he could easily approximate the result. Here we offer a possible way. Refer again to Figure 3. In Eratosthenes' time there was available the following pseudo-trigonometric relation:

$$\frac{\beta}{\alpha} > \frac{AU}{AE}.$$

This relation, which is equivalent to $\beta/\alpha > \sin\beta/\sin\alpha$, is used in Aristarchus's *On the Sizes and Distances of the Sun and the Moon*.[36] Since $\alpha = \gamma - \beta$, this may be written as

$$\beta > \frac{\gamma}{\left(1 + \dfrac{AE}{AU}\right)}.$$

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[35] Noel Swerdlow, "Hipparchus on the Distance of the Sun," *Centaurus*, 1969, *14*:287–305; and G. J. Toomer, "Hipparchus on the Distances of the Sun and Moon," *Arch. Hist. Exact Sci.*, 1974, *14*:126–142.

[36] Aristarchus uses this inequality in proposition 4. See J. L. Berggren and Nathan Sidoli, "Aristarchus's *On the Sizes and Distances of the Sun and the Moon*: Greek and Arabic Texts," *Arch. Hist. Exact Sci.*, 2007, *61*:213–254. Aristarchus is also familiar with the ancient equivalent of $\beta/\alpha < \tan\beta/\tan\alpha$ (used in propositions 4 and 11), which, applied to our situation, would give $\beta/\alpha < SU/ES$.

From Figure 3, we see that $AU > SU$, so it is still the case that

$$\beta > \frac{\gamma}{\left(1 + \dfrac{AE}{SU}\right)}.$$

Then, putting $AE = R$ and $SU = d_\odot - R$, we have, finally,

$$\beta > \gamma\left(1 - \frac{R}{d_\odot}\right).$$

This gives a minimum possible value for β. With $d_\odot/R = 101.7$ we find $\beta_{min} = 7.12920°$. β_{max} is of course just γ (= 7.2°); this case would apply if the Sun were at infinity. The circumference of the Earth is in either case calculated simply as $C = 5{,}000$ stades $\times$ $360/\beta$. With β_{max} we of course obtain $C_{min} = 250{,}000$ stades. And with β_{min} we get $C_{max} = 252{,}483$ stades, negligibly different from the value obtained above by trigonometric calculation.

<div align="center">ON ERATOSTHENES' ASSUMPTIONS</div>

Some further insight may be gained into Eratosthenes' procedure by examining the parallaxes that result from his solar and lunar distances.[37] The implied horizontal parallaxes are $P_\odot = \sin^{-1}(40{,}107/4{,}080{,}000) = 0.563°$ and $P_{\mathbb{C}} = \sin^{-1}(40{,}107/780{,}000) = 2.947°$. The total parallax $P_\odot + P_{\mathbb{C}}$ is 3.510°.

Now, in the procedure based on the lunar eclipse diagram of Aristarchus of Samos, it is required that $P_\odot + P_{\mathbb{C}} = \sigma + \tau$, where σ is half the angular diameter of the Sun and τ is half the angular diameter of the Earth's shadow.[38] This relation is not stated by Aristarchus but follows from his diagram. As is well known, Aristarchus assumed that the angular diameters of the Sun and Moon are equal (since the Moon just covers the Sun during a total solar eclipse), that both are equal to 2°, and that the angular diameter of the Earth's shadow is equal to two Moons. Thus Aristarchus has $\sigma = 1°$ and $\tau = 2°$, so the total parallax amounts to 3° (which is about a threefold overestimate).

Eratosthenes's total parallax is about 3.5°, so it seems plausible that he began with Aristarchus's assumptions, then perhaps deliberately assumed a slightly larger total parallax, perhaps by making the Earth's shadow a bit larger (which would be an improvement). If we suppose that Eratosthenes took the Earth's shadow to be 2½ Moons wide and kept the angular diameter of the Sun or Moon at 2°, we would indeed have $\sigma + \tau = 3.5°$.[39] Whether Eratosthenes proceeded thus, we cannot really know. But his solar and lunar distances do show that his original assumptions for σ and τ could not have been very

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[37] Let the Moon be on the horizon for an observer located at the Earth's surface. The Moon's horizontal parallax is the angle between the lines of sight to the Moon for this observer and for an imaginary observer at the center of the Earth.

[38] See, e.g., James Evans, *The History and Practice of Ancient Astronomy* (New York: Oxford Univ. Press, 1998), pp. 69–70.

[39] It is interesting that Hipparchus took the diameter of the Earth's shadow to be 2½ Moons, when the Moon is at its mean distance and new or full. See Ptolemy, *Almagest* 4.9; and G. J. Toomer, *Ptolemy's "Almagest"* (London: Duckworth, 1984), p. 205.

different from those of Aristarchus. It may seem surprising that his assumed angular diameters of the Sun and Moon should not have improved on Aristarchus's notorious overestimate of 2°. But if Eratosthenes were attempting to establish an upper limit for the circumference of the Earth it would have made good sense to assume the largest possible values for σ and τ, for this results in the closest possible Sun and the largest Earth.

Of course, Eratosthenes must have replaced Aristarchus's assumption about the lunar dichotomy (that the angle between the Sun and the Moon at quarter Moon is 87°) with something else entirely. This is the assumption of Aristarchus that leads to $d_\odot/d_{\mathbb{C}} \cong 19$. Since Eratosthenes found $d_\odot/d_{\mathbb{C}} \cong 5.2$, he could not have relied on this assumption.

A CONJECTURE CONCERNING MACROBIUS

We are now in a position to say something about Macrobius's claim that Eratosthenes' measure of the Sun was 27 times his measure of the Earth—though here things must become more conjectural. We need not put much faith in the technical competence of this late Latin compiler, but let us see whether any sense can be made of this number. Eratosthenes' small solar and lunar distances naturally result in small values for the linear diameters of the Sun and Moon. For example, if we take the angular diameter of the Sun to be ½° (close to reality) and assume the attested distance of 4,080,000 stades, it actually results in a Sun that is *smaller* than the Earth. However, as we showed above, Eratosthenes' solar and lunar parallaxes imply that he used angular diameters that are close to the 2° assumed by Aristarchus of Samos.[40]

Assuming, therefore, $d_\odot = 4,080,000$ stades and a solar angular diameter of 2°, we find that $R_\odot/R = 1.775$. Then $(R_\odot/R)^2 = 3.15$. That is, the *surface areas* of the Sun and the Earth stand in a three-to-one ratio. For a geographer, the surface of a thing may well have been its most fundamental aspect—it is a measure of the number of mountains, rivers, and seas that may be squeezed in. We conjecture that someone later misinterpreted the 3:1 ratio as a statement about *linear measure*. Subsequently, this was cubed to obtain a mistaken value of 27 for the ratio of the volumes. Alternatively, we cannot exclude the possibility that Macrobius's 27 is simply nonsense.

CONCLUSIONS

Eratosthenes' calculation of the circumference of the Earth, as recounted by Cleomedes, has always seemed anticlimactic. It was made more than a century after the earliest Earth measurements. Moreover, it is methodologically far simpler than the measurement of the sizes and distances of the Sun and Moon by Aristarchus of Samos earlier in Eratosthenes' own century and makes no use of Aristarchus's advance. As we have seen, Eratosthenes dedicated a whole book to the subject: bits and pieces of this book are preserved, and it must have amounted to something far more interesting than the little calculation presented by Cleomedes. Finally, there is the mystery of the second, slightly larger value for the circumference of the Earth (252,000 stades rather than 250,000) attributed to Eratosthenes by many Greek and Roman sources.

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[40] The lunar distance alone, for which the textual tradition is much more secure, is enough to force us to this conclusion. As we have seen, Eratosthenes' lunar parallax alone is nearly 3°. We must then have $\sigma + \tau > 2.947°$; so, assuming that the angular sizes of the Sun and Moon are equal, and taking $\tau = 2\sigma$ (as in Aristarchus), we find that the angular diameter of the Sun must be greater than 1.96°.

In this essay we have shown that all these oddities may easily be reconciled. The figure of 250,000 stades is Eratosthenes' lower-limit result, obtained by the method described by Cleomedes and based on the assumption that the Sun is infinitely distant. But Eratosthenes, continuing with the program of Aristarchus of Samos, also examined the consequences for the size of the Earth if the Sun were at a finite distance. The figure of 252,000 stades is his upper-limit value for the circumference, based on the assumption that the Sun is 4,080,000 stades from us. Moreover, we can now understand why Eratosthenes treated the distances of the Sun and the Moon in a work supposedly devoted to the measurement of the Earth. The distances of the Sun and Moon are entangled, for example, in Aristarchus's method and must be found together. And the distance of the Sun is required for Eratosthenes' calculation of the upper limit for the size of the Earth.

Eratosthenes' book *On the Measurement of the Earth* therefore contained something original and took a step forward. Rather than treating the size of the Earth and the distance of the Sun as geometrically separate problems, as everyone had before him, he showed that they are related. And he showed, too, that any plausible finite distance for the Sun will make only a very minor difference in the circumference of the Earth. This is why, for example, Hipparchus could accept his value of 252,000 stades for the circumference of the Earth: any plausible solar distance (plausible from the point of view of an astronomer of the second century B.C.E.) would result in a value not terribly far from this. And, on the other hand, 250,000 stades was definitely too small, simply because it was predicated on the Sun being at an infinite distance.